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Determination of Fiber Proof-Test Stress for Undersea Lightguide Cable

By T. C. CHU and H. C. CHANDAN*

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A method is developed to determine the fiber strength requirements for the undersea lightguide cable. This method utilizes the theories involved in fiber proof testing, undersea cable dynamics, and nonlinear tensile behavior of cable. The method is illustrated with a sample cable currently being developed for the transatlantic system. This method provides great flexibility in designing undersea lightguide cable. It can be applied to justify a new cable design at an early stage or to select proper fiber proof-test levels for an existing cable under different operating conditions, i.e., deeper ocean, worse sea state, or faster recovery.

I. INTRODUCTION

Because of the difficulty and high cost of repair, undersea cable requires high reliability. Operations such as laying, recovery, and holding impose high tensions on the cable.[1] The high tension causes high strain and permanent deformation in the cable, and it may reduce or impair system reliability. Much experience has been gained over the past decades in designing conventional undersea cable with metallic conductors to withstand the high tension. In general, cable strains well in excess of 1 percent can be tolerated for the coaxial cables with metallic conductors due to the ductile nature of metals.

For the new undersea cable that has lightguides as the signal-carrying medium, the problems of high tension and strain cause greater

* Authors are employees of AT&T Bell Laboratories.

concerns because of the static and dynamic fatigue of optical fibers.[2,3] To ensure that the fibers survive the high-tension cable operations (laying, recovering, and holding) and last for 25 years, the entire length of each fiber is proof tested[2,3] to a given stress before cabling. The proof-test level is chosen by taking into account the load-time history of the fibers during the entire service period. An appropriate proof-test level must be chosen since an underestimated proof-test stress reduces the system reliability, and an overestimated one increases system cost due to reduced fiber yield or increased fiber loss caused by more frequent repair splices during proof testing.

In this paper a method is developed to determine the fiber proof-test stress for undersea lightguide cable. In Section II the theory of proof testing is discussed, with special attention to the material constants used in the calculations. In Section III the cable tensions as function of time are analyzed. In Section IV the nonlinear tensile property of the cable is discussed and is used to obtain the cable and fiber strain. The fiber strain is then used to determine the fiber proof-test stress. In Section V the method is applied to a sample cable that is currently under development for transatlantic systems.

II. PROOF-TEST LEVEL

It is well established that the strength of glass optical fibers is determined by micrometer- or submicrometer-size surface or subsurface flaws. These flaws grow with time under the combined action of tensile stress, temperature, and ambient moisture. These fracture-initiating flaws are randomly distributed along the length of the fiber. Therefore, the entire length of the fiber is proof tested to screen out flaws larger than those that correspond to the proof-test level. For example, the largest initial flaw present in a given length of 200 kpsi, proof-tested, fused-silica fiber is 0.212 μm.[4]

Subsequently, the proof-tested fiber is subjected to varying stresses for different time periods during cable manufacture, laying, recovery, and repair. During these operations, the preexisting flaws in the fiber grow, and this flaw growth is cumulative. At present, no commercial scheme is available that will prevent crack growth in fibers. Therefore, we choose the proof-test level that restricts crack growth during expected worst-service conditions to yield a desired design life for the fibers. In other words, by proof testing we ensure that the initial crack size is sufficiently small so that the full range of stresses to which the fiber is subjected in service will not cause the fiber to rupture.

Assuming a power law for crack growth, the largest crack, a_p, in a proof-tested fiber will grow at a rate $(da)/(dt)$ given by[5]

$$\frac{da}{dt} = AK_I^N, \tag{1}$$

where K_I is the stress-intensity factor (the subscript I refers to the crack-opening mode) at the crack tip and is given by

$$K_I = Y\sigma a^{1/2}, \tag{2}$$

where Y is a geometrical factor related to the shape of the crack and the specimen ($\simeq$1.241 for semicircular part-through crack, normal to the axis of a cylindrical fiber); σ is the applied stress. The characteristic crack-growth parameters A and N are coupled and are empirically determined from dynamic and static fatigue tests.[6] From a collection of experimental data covering several independent investigations, we have observed that the relationships between $\log A$ and N are different for dynamic and static fatigue situations. For short-term loads A_d, increasing with time, our studies have indicated that $\log A_d = 3.22N - 8.516$. For a static load A_s, $\log A_s = 3.289N - 10.05$. The choice of N depends on the Relative Humidity (RH) in the ambient environment: $N = 14$ to 16 for wet environment (97-percent RH) and $N = 17$ to 20 for typical laboratory environment (40- to 60-percent RH).

From eqs. (1) and (2), an applied stress of σ_1 for a time t_1 will cause the crack to grow from the proof-test level of a_p to a_1 and can be calculated by

$$a_1^{-\frac{N}{2}+1} - a_p^{-\frac{N}{2}+1} = \frac{2-N}{2}\int_0^{t_1} AY^N[\sigma_1(t)]^N dt, \tag{3}$$

where a_1 is the flaw size after the fiber has been under an applied stress $\sigma_1(t)$ for a time t_1. The fused silica optical fibers behave in a linear elastic manner up to the breaking point; therefore, the stress $\sigma_1(t)$ in eq. (3) can be expressed in terms of strain, $\epsilon_1(t) = [\sigma_1(t)]/E$, where E is the Young's elastic modulus of the fiber ($E = 71.9$ GPa or 10.4×10^6 psi). Furthermore, when the fibers are subjected to a sequence of stress loadings, the flaw growths are additive.[4] From these considerations, eq. (3) can be rewritten as

$$a_p^M = a_f^M - \sum_{i=1}^{Q} AME^NY^N \int_0^{t_i} [\epsilon_i(t)]^N dt, \tag{4}$$

where $M = (2 - N)/2$ and a_f is the final flaw size after the sequence of stress loadings. If the flaw growth is considerable, say $a_f/a_p \geq 10$, and we assume that N is in the range of 14 to 20 ($M \approx -7$), then

$$\frac{a_f^M}{a_p^M} \approx 10^{-7}.$$

Thus, the term a_f^M can be neglected and

$$a_p^M \cong \sum_{i=1}^{Q} (-AM)E^N Y^N \int_0^{t_i} [\epsilon_i(t)]^N dt. \tag{5}$$

The required proof-test level is determined from the fracture mechanics relation,

$$\sigma_p = \frac{K_{IC}}{Y a_p^{1/2}}, \tag{6}$$

where K_{IC} is the fracture toughness of the material. For fused silica fibers,

$$K_{IC} = 0.789 \text{ (unit in MPa } \sqrt{m}),$$

and as previously mentioned, $Y \cong 1.241$. Hence,

$$\sigma_p = \frac{0.6357}{a_p^{1/2}}, \tag{7}$$

where a_p is determined from eq. (5) in meter and σ_p is in MPa. Thus, the fibers proof tested at σ_p will endure the stress loadings during cable laying, recovery, repair, and long-term service without fracture.

III. CABLE TENSIONS DURING THE OPERATIONS

To evaluate the fiber proof-test stress, the fiber strain ϵ_f in eq. (5) has to be determined. The amount of fiber strain is related to the cable tension, cable tensile property—i.e., tension versus strain relation—and the coupling between the fibers to the cable structure. In a cable with its fibers fully coupled to the cable structure, the fiber strain is equal to the cable strain, and this is the worst case. By knowing the cable tensions as functions of time during each operation, the cable strain as a function of time can be obtained from the cable tensile property. The cable tension as a function of time can be derived from the maximum tension at the ship, as shown below.

3.1 Laying

In a normal cable-laying operation, the cable forms a straight-line configuration, as shown in Fig. 1a. The cable tension at the ocean bottom is theoretically equal to zero. The cable tension at the ship is equal to wh,[1] where w is the cable weight per unit length in seawater and h is the ocean depth. By following a cable element during the laying operation, the time-dependent tension experienced by the element can be found as

$$T(t) = wh - \frac{wh}{\left(\frac{h}{v_l \sin(\alpha_s)}\right)} t, \qquad 0 \le t \le \frac{h}{v_l \sin(\alpha_s)}, \tag{8}$$

where v_l is the cable-laying speed, t is the elapsed time after the cable element leaves the ship and enters the water, and α_s is the angle between the cable and sea level and is

$$\alpha_s = \frac{H}{v_l},$$

where H is the hydrodynamic constant of the cable. For most undersea cable used in deep water, H ranges between 30 to 50 degree-knots. The cable-laying speed is usually about 7 to 8 knots, thus, the angle α_s is less than 10 degrees. For such small angles, the contributions of wave motion and repeater weight to the tension are negligible. Equation (8) is sketched in Fig. 2a.

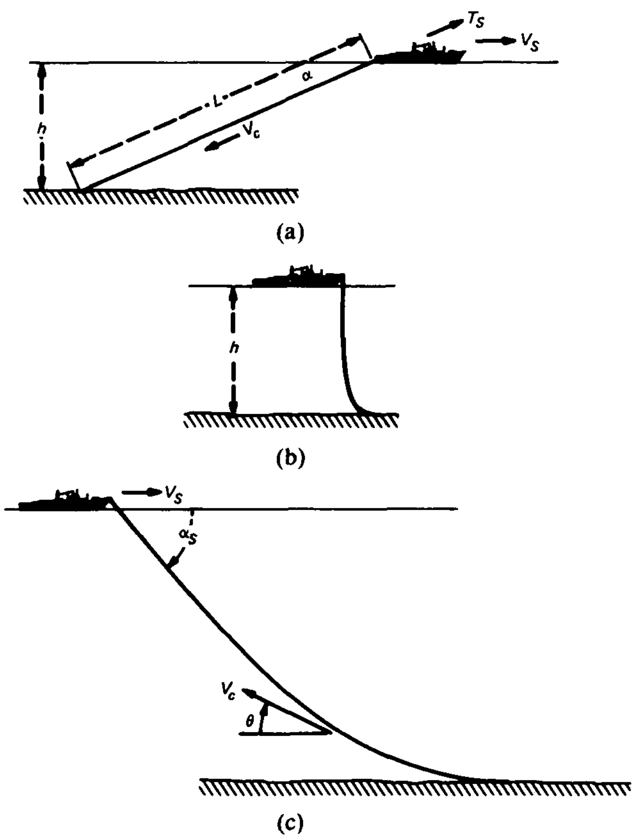


Fig. 1—Cable operations. (a) Laying. (b) Testing or splicing. (c) Recovery.

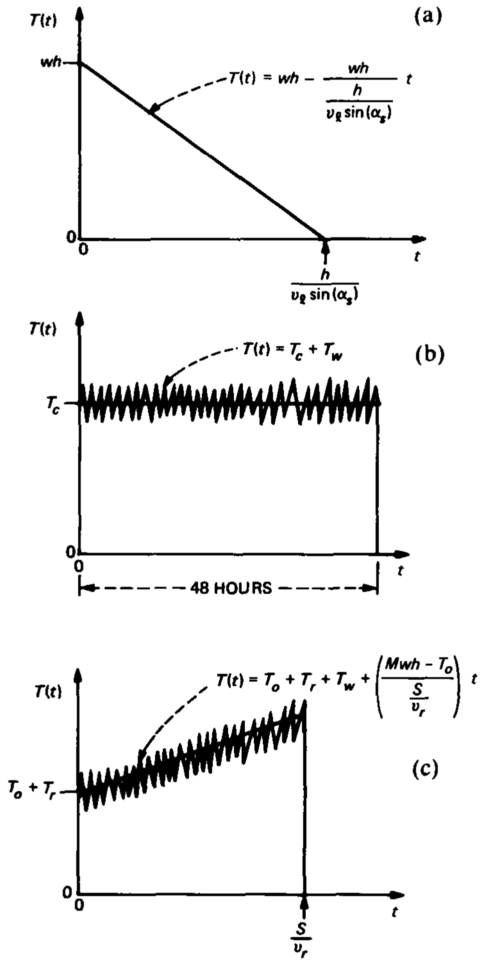


Fig. 2—(a) The time-dependent tensions experienced by the cable element during the laying operation. (b) Upper bound for cable tension during testing. (c) Catenary formed by the cable during recovery.

3.2 Recovery

During recovery, the cable forms a catenary, as shown in Fig. 1b, and the length of the catenary S can be evaluated.[1] For a given recovery speed v_r, the recovery time of a cable element is given by

$$t = \frac{S}{v_r}.$$

Since the cable forms a catenary, the cable tension at the ocean bottom, T_0, is not zero. The cable tension at the ship, not including the effects of wave motion and repeater weight, is $\bar{M}wh$, where $\bar{M}$ is a magnification factor that indicates the difference in tension between laying and recovery. The factor $\bar{M}$ has the following form:[1]

$$\bar{M} = \frac{1}{1 - \tan^2 \frac{\alpha}{2} \left(\frac{\cos \alpha + \cos \alpha_s}{1 - \cos \alpha \cos \alpha_s} \right)^{\frac{1}{\gamma}}},$$

$$\gamma = \frac{\sin^2 \alpha}{2 - \sin^2 \alpha},$$

where α_s is the angle between the cable and sea level and α is the ratio between the cable hydrodynamic constant H and the recovery speed v_r. Assuming that the tension experienced by a cable element during recovery varies linearly with time, the cable tension as a function of time—including the wave motion and repeater weight—turns out to be

$$T(t) = T_0 + \left(\frac{\bar{M}wh - T_0}{\frac{S}{v_r}} \right) t + T_w + T_r,$$

$$0 \leq t \leq \frac{S}{v_r}, \tag{9}$$

where T_r is the weight of a cable-carried point mass (such as a repeater) in seawater, and T_w is the additional tension due to the wave motion. T_w is expressed by

$$T_w = \sqrt{\overline{EA}\rho_c} v_y \cos \frac{2\pi t}{\tau} \cos \alpha_s,$$

where $\overline{EA}$ is the cable tensile stiffness, ρ_c is the cable mass per unit length, v_y is the maximum ship vertical velocity, τ is the period of wave motion, and α_s is the recovery angle. In normal recovery, the angle α_s is kept close to 90 degrees to reduce the tension, thus cos $\alpha_s \simeq 1$. Equation (9) is expressed in Fig. 2b.

3.3 Holding

After its recovery, the cable is held for a period of time for testing and repair. At present, the required splicing time for the lightguide cable is estimated at 16 hours. Assuming (pessimistically) that three attempts are made before a successful repair is executed, then the total holding time is 48 hours. During holding period, the cable also forms a catenary that is essentially stationary except for the effect of wave motion, as shown in Fig. 1c. The tension experienced by the cable element at the ship can be shown as

$$T(t) = T_c + T_w \qquad 0 \le t \le t_0, \tag{10}$$

where t_0 is the holding time, T_w is the additional tension due to the wave motion, T_c is the tension due to the catenary and can be evaluated from the theory of a catenary as[7]

$$T_c = wC \cos h(z),$$

where w is the cable weight per unit length in water, and C and z are constants to be determined from the ocean depth h and the cable angle α_s at holding. Because the cable sections suspended near the ocean bottom experience lower tension than the cable elements near to the ship while holding, the survival of cable elements at the ship guarantees the survival of the remaining cable. In short, $T(t) = T_c + T_w$ expresses the upper bound for cable tension while loading. This equation is represented in Fig. 2c.

IV. CABLE TENSILE PROPERTIES

By knowing the cable tensions during the operations, the cable strain can be found if the tensile property of the cable is known. A method has been developed to accurately predict the tensile property of undersea cable in both elastic and plastic regions.[8] The method relates the cable tension T to cable strain ϵ_c by a fifth-degree polynomial as

$$T = C_0 + C_1\epsilon_c + C_2\epsilon_c^2 + C_3\epsilon_c^3 + C_4\epsilon_c^4 + C_5\epsilon_c^5,$$

where C_0 through C_6 are constants depending on the mechanical properties of the constituent cable components. An example of such a polynominal is shown in Fig. 3. Since our present need is to find the

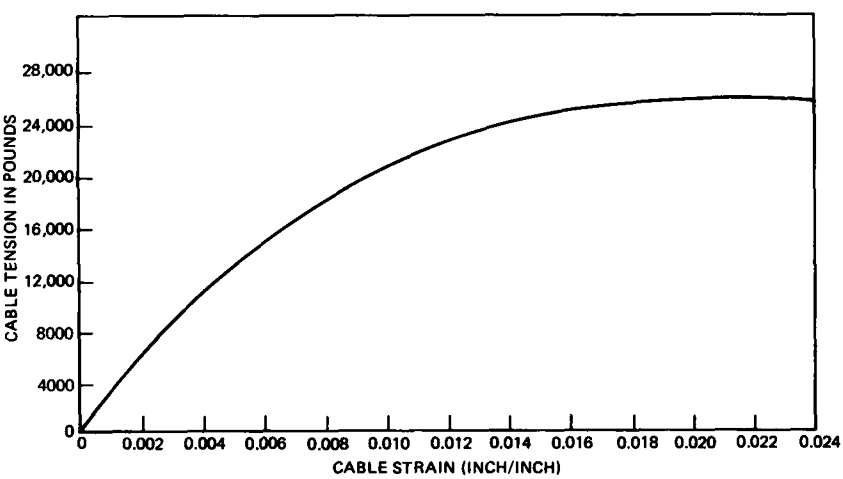
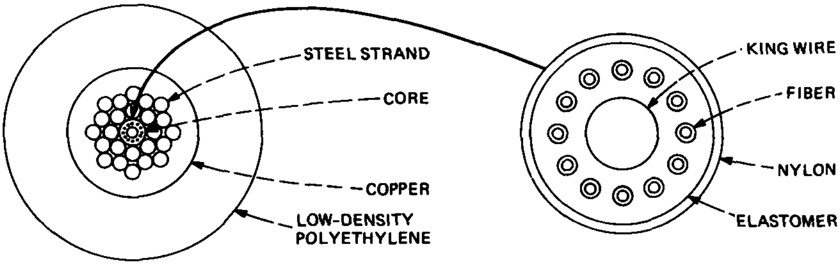


Fig. 3—Tensile properties of the TAT-8 transatlantic cable.



CABLE STRUCTURE:

STRAND DIAMETER = 9.47 mm (0.373 in.)
CONDUCTOR OD (COPPER) = 10.46 mm (0.412 in.)
INSULATION OD = 21 mm (0.827 in.)

CABLE CORE:

KING WIRE OD = 0.71 mm (0.028 in.)
NUMBER OF FIBERS = 12
FIBER OD (COATED) = 250 μm (0.010 in.)
NYLON THICKNESS = 0.1 mm (0.004 in.)
CORE OD = 2.97 mm (0.117 in.)

Fig. 4—Cable structure and dimensions for the TAT-8 transatlantic cable.

cable strain for a given tension, an inverse function of the above relation is required. This can be done by the numerical regression method. The resulting relation is

$$\epsilon_c = \bar{C}_0 + \bar{C}_1 T + \bar{C}_2 T^2 + \cdots + \bar{C}_k T^k, \tag{11}$$

where k and $\bar{C}_0$ through $\bar{C}_k$ are determined with desired accuracy from the numerical regression.

Since the cable tension during its operations is a function of time $T(t)$, eq. (11) gives the cable strain as a function of time, i.e., $\epsilon(t)$. If the fibers are fully coupled to the cable structure, the fiber strain is equal to the cable strain $\epsilon(t) = \epsilon_c(t)$, and this is the strain required in eq. (5) to evaluate the fiber proof-test level.

V. APPLICATIONS

This method of testing fiber strength is applied to the cable that is currently being developed for the TAT-8 transatlantic system. The cable construction and dimensions are shown in Fig. 4. The cable and fiber properties required in the proof-test stress evaluation are listed in Table I. The cable tension during each operation is evaluated under realistic operating conditions.

Table I—Cable and fiber properties

Cable weight per unit length in seawater w	0.326 lb/ft
Cable hydrodynamic constant H	43.5 degree-knots
Cable mass per unit length ρ_c	$0.0174 \frac{\text{lb} - \text{s}^2}{\text{ft}^2}$
Cable tensile stiffness $\overline{EA}$	2.9×10^6 lb
Fiber "N" number	17
Fiber tensile modulus E	10.4×10^6 psi

5.1 Laying

Following are the conditions under which cable was laid:

1. Laying speed $v_l = 7$ knots
2. Ocean depth $h = 18{,}000$ ft (transatlantic installation)
3. Cable angle $\alpha_s = H/v_l = 6.22$ degrees
4. Cable touchdown time $= h/[v_l \sin(\alpha_s)] = 3.93$ hours.

Considering the above conditions and the cable properties listed in Table I, eq. (8) becomes

$$T(t) = 5875 - \frac{5875}{3.93 \times 60 \times 60}\, t$$

$$0 \le t \le 3.90 \times 60 \times 60 \text{ seconds.} \qquad (12)$$

5.2 Recovery

Following are the conditions under which the cable was recovered:

1. Recovery speed $v_r = 1$ knot
2. Recovery angle $\alpha_s = 85$ degrees
3. Vertical ship velocity due to wave motion $v_y = 17$ ft/s (sea state 6)
4. Ocean depth $h = 18{,}000$ ft
5. $wh = 5875$ lb
6. Additional tension due to wave motion $T_w = 3829$ lb. (To simplify the integration, the tension due to the wave motion is assumed to be equal to $\sqrt{EA\rho_c}v_y$, the amplitude of the oscillating tension. This simplification results in a more conservative fiber proof-test stress.)
7. $\bar{M}wh = 13{,}488$ lb
8. Cable bottom tension $T_0 \equiv \bar{M}wh - wh = 7613$ lb
9. Suspended cable length[1] $= 1.65 \times 18{,}000 = 29{,}700$ ft
10. Recovery time $t = S/v_r = 4.89$ hours
11. Repeater weight in seawater $T_r = 1000$ lb (presently, the repeater weight has been reduced to 300 lb in water).

With the above conditions and cable properties, eq. (9) becomes

$$T(t) = (7613 + 3829 + 1000) + 0.33373t$$

$$0 \le t \le 4.89 \times 60 \times 60 \text{ seconds.} \qquad (13)$$

5.3 Holding

Following are the conditions during holding:

1. Ocean depth $h = 18{,}000$ ft
2. Cable angle at holding $\alpha_s = 75$ degrees
3. Additional tension due to wave motion $T_w = 3829$ lb
4. Cable weight per unit length in water $= 0.326$ lb/ft.

By knowing the ocean depth h and cable angle α_s, the constant C and z can be evaluated from the catenary theory as

$$C = 6299.5 \text{ ft}$$

$$z = 2.0563$$

$$T_c = WC \cos h(z) = 8133 \text{ lb.}$$

Thus, eq. (10) becomes

$$T(t) = 8133 + 3829$$

$$0 \le t \le 48 \times 60 \times 60 \text{ seconds.} \quad (14)$$

5.4 Cable tensile properties

The theoretical tensile property of the submarine cable evaluated by the method previously discussed is shown in Fig. 4.[8] The inverse polynominal is found to have the following form:

$$\epsilon = \bar{C}_0 + \bar{C}_1 T + \bar{C}_2 T^2 + \bar{C}_3 T^3 \quad (15)$$

for $0 \le T \le 21{,}300$ lb, where

$$\bar{C}_0 = -2.9758111668 \times 10^{-5}$$

$$\bar{C}_1 = +3.96935593146 \times 10^{-7}$$

$$\bar{C}_2 = -8.1189437448 \times 10^{-12}$$

$$\bar{C}_3 = +6.40675624028 \times 10^{-6}$$

and the standard error $\chi = 6.9 \times 10^{-5}$. It is noted that eq. (15) applies for cable tension below 21,300 lb. It is adequate because the maximum tension expected by the cable during its operations is about 17,500 lb.

5.5 Proof-test stress evaluation

The cable tension $T(t)$ in each of eqs. (12), (13), and (14) is substituted into eq. (15) to obtain $\epsilon_1(t)$, $\epsilon_2(t)$, and $\epsilon_3(t)$. Substitution of $\epsilon_1(t)$, $\epsilon_2(t)$, and $\epsilon_3(t)$ into eq. (6) for $\epsilon_i(t)$ gives, upon numerical integration, the initial flaw size a_p:

$$a_p^{-7.5} = \underset{\text{(laying)}}{5.5685763 \times 10^{39}} + \underset{\text{(recovery)}}{5.139996 \times 10^{48}} + \underset{\text{(holding)}}{5.0337450 \times 10^{45}}$$

or

$$a_p = 3.2 \times 10^{-7} \text{meter} = 0.32\ \mu\text{m}.$$

The corresponding proof-test stress is obtained from eq. (7):

$$\sigma_p = 1.124 \text{ GPa} = 163 \text{ kpsi}.$$

Thus, a proof-test stress greater than 163 kpsi should guarantee the survival of the fibers in the cable. Generally, however, a somewhat higher proof-test level (200 kpsi) is specified for transatlantic cable to ensure that adequate margin is provided to offset any subtle effects brought about by the cable manufacturing and handling processes that are not covered by the model.

VI. CONCLUSIONS

A method is developed to evaluate the required proof-test stress for the fibers used in the undersea cable. The method provides great flexibility in designing the cable. It can be applied to justify a cable design and verify the adequacy of a fiber proof-test level at an early stage, or to select proper fiber proof-test levels for an existing cable under different operating conditions, i.e., deeper ocean, worse sea state, or faster recovery.

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AUTHORS

Harish C. Chandan, B. Tech. and M. Tech. (Metallurgical Engineering), in 1969 and 1971, respectively, Indian Institute of Technology, Kanpur, India; M.S. (Solid State Science), 1974, Syracuse University; Ph.D. (Ceramics Science), 1977, Pennsylvania State University; AT&T Bell Laboratories, 1977—. Since joining AT&T Bell Laboratories, Harish has worked in the area of strength, fatigue, and reliability of optical fibers. He is currently involved in dehydration and sintering of soot preforms for optical fibers made by the outside deposition process. Member, American Ceramic Society, National Institute of Ceramic Engineers.

Tek-Che Chu, B.S., 1964, Cheng Kung University (Taiwan); M.S., 1967, Syracuse University; Ph.D., 1971 (Aerospace Engineering), Cornell University; AT&T Bell Laboratories, 1972—. Mr. Chu has worked on the physical design of T4M digital transmission system and the development of an optical fiber connector. Since 1981 he has been working on the design of undersea lightguide cable.