BELLCOMM, INC.
935 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D.C. 20024

COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- Axisymmetric Shells

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DATE- July 24, 1969

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AUTHOR(S)- S. Kaufman

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(ASSIGNED BY AUTHOR(S)-

ABSTRACT

This report is a combination theoretical and user manual for the digital program AXSHEL. AXSHEL solves for influence and stiffness matrices of shells of revolution by a finite element technique. Bending as well as mebrane strain energy has been included in the formulation. An option to include the effects of differential stiffness (initial internal forces) has been incorporated.

Another option of AXSHEL is the solution of internal forces and deflections due to pressure and/or concentrated static loadings. Included is the results of a test run of a spherical cap subjected to an external pressure. Comparisons are made to analytic theory.

The impetus for the development of this program came from the S-II Pogo Analysis Group at Bellcomm. More specifically, a need arose to obtain a more exact representation of the stiffness characteristics of the S-II LOX tank shell.

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I. INTRODUCTION

A digital program AXSHEL has been written for the UNIVAC 1108 to solve the static response characteristics of axisymmetric shells. These responses include: 1) stresses, internal forces and deflections due to pressure and/or concentrated loadings, 2) the internal load influence matrix, 3) the deflection influence matrix and 4) the stiffness matrix. The method of solution is based on the matrix force method. Presently, the program is limited to approximately 20 stations (nodal circles) with one axial constraint. Shell thickness may vary from station-to-station. Radial constraints are permitted at the nodal circles as required. Meridianal rotational (bending) constraints are permitted at terminal stations. only. All constraints may be either rigid or flexible as desired. Strain energy of both flexure and membrane has been incorporated in the program. Algorithms for differential stiffness are included.

For a test problem a 39° spherical cap with fixed support (as shown in Figure 1) was subjected to an external pressure of 284 psi. The meridianal (ϕ) and hoop (θ) stresses resulting from this loading are compared with analytic theory in Figure 2. The maximum meridianal stress computed (-7750 psi) compared within 5% with the theory (-8100 psi). The spring constraints k_u and k_ϕ in Figure 1 refer to the radial force and meridianal moment constraint at station 1. These constraints replace the elastic properties of an equivalent circular disc discussed in section III of this report.

II. EQUILIBRIUM

The method as presented in this report begins with a definition of nodal circles (N1 of them) and force elements connecting these circles. Axial and radial force equations of equilibrium are written for each nodal circle. In addition, two tangential moment equations of equilibrium are formed, one for each terminal nodal circle. The force elements are divided

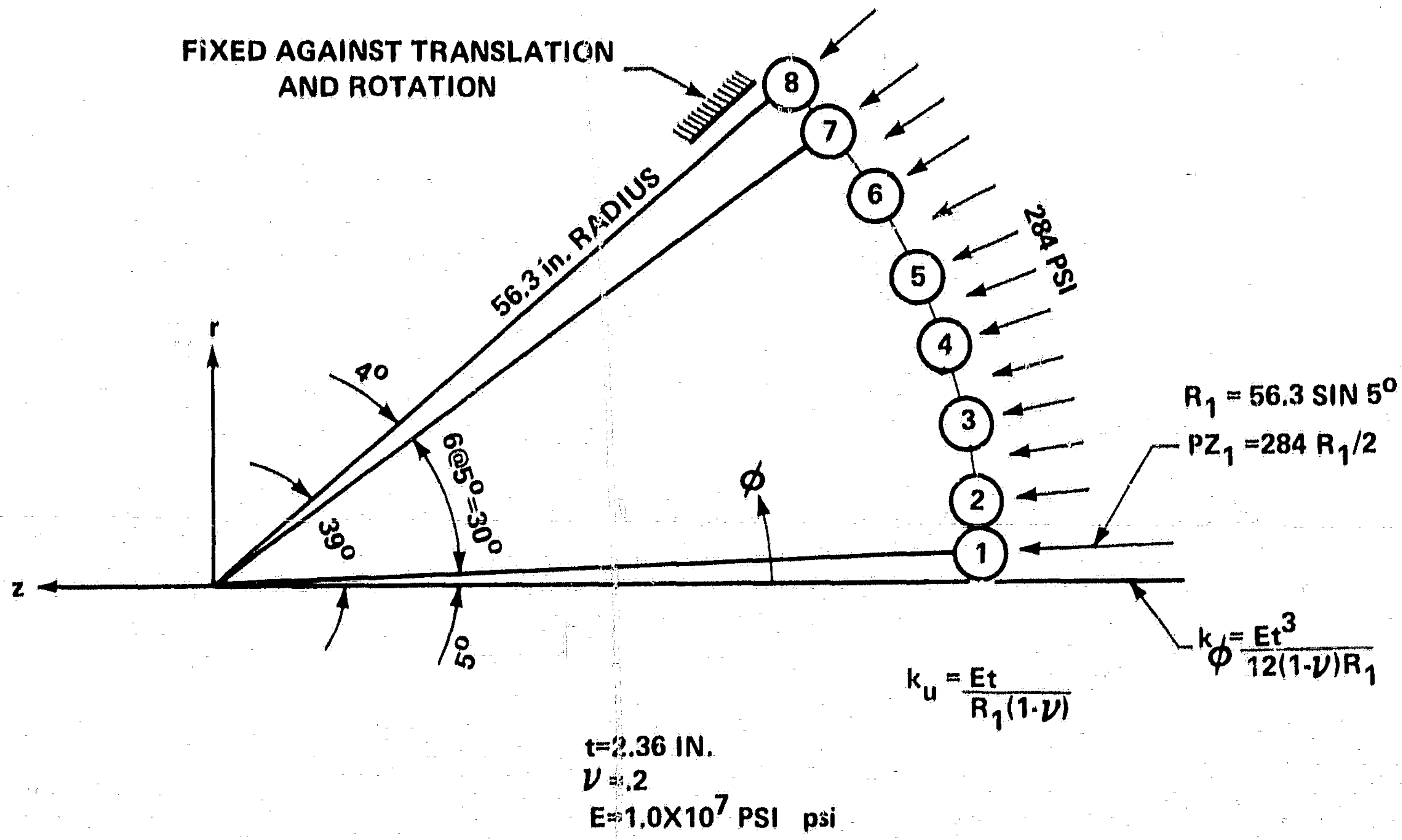
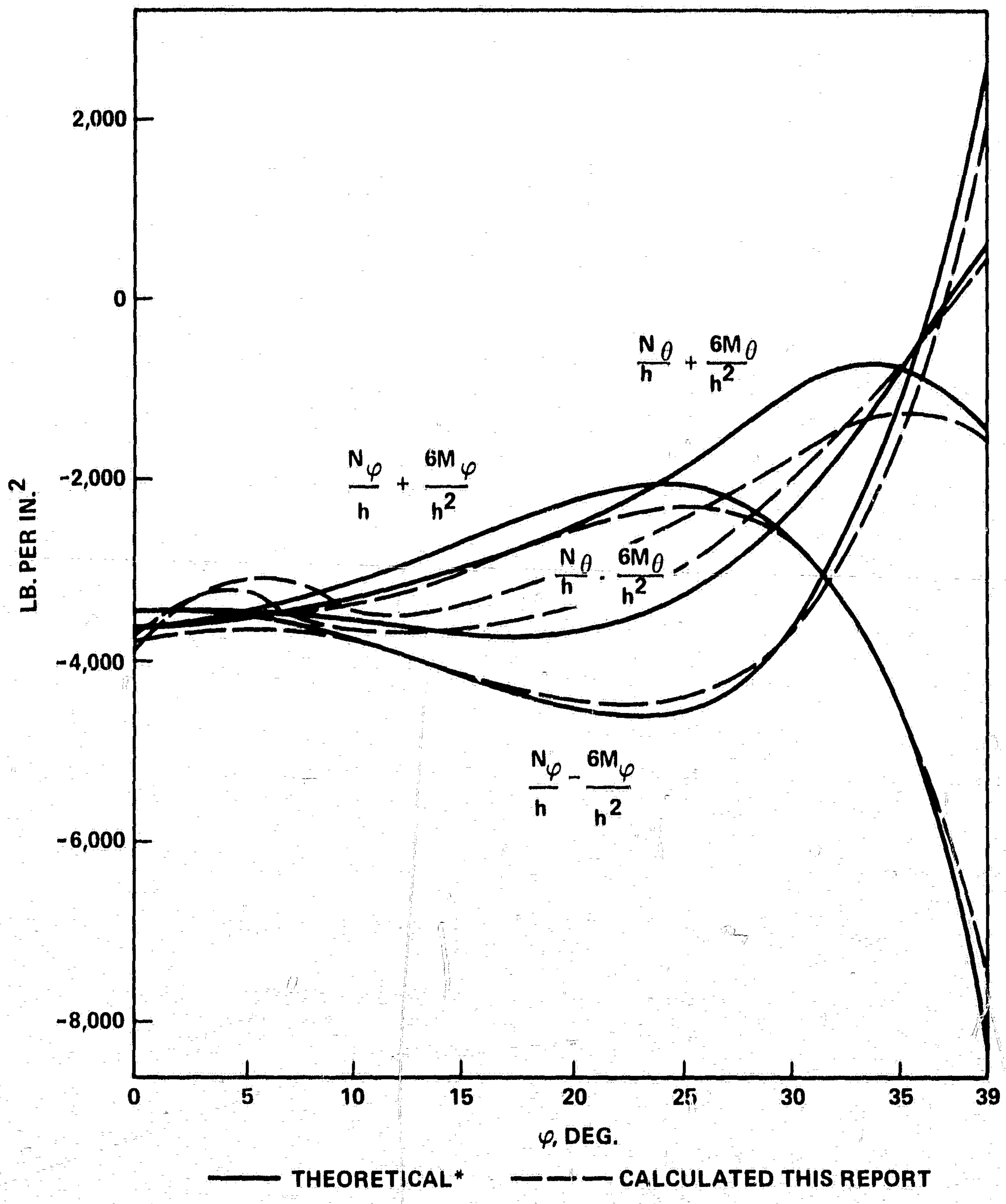


FIGURE 1 TEST PROBLEM - 39° SPHERICAL CAP



* "THEORY OF PLATES AND SHELLS," S. TIMOSHENKO AND S. WOINOWSKY-KRIEGER, 2ND EDITION, McGRAW-HILL, P.545 (FIGURE 270)

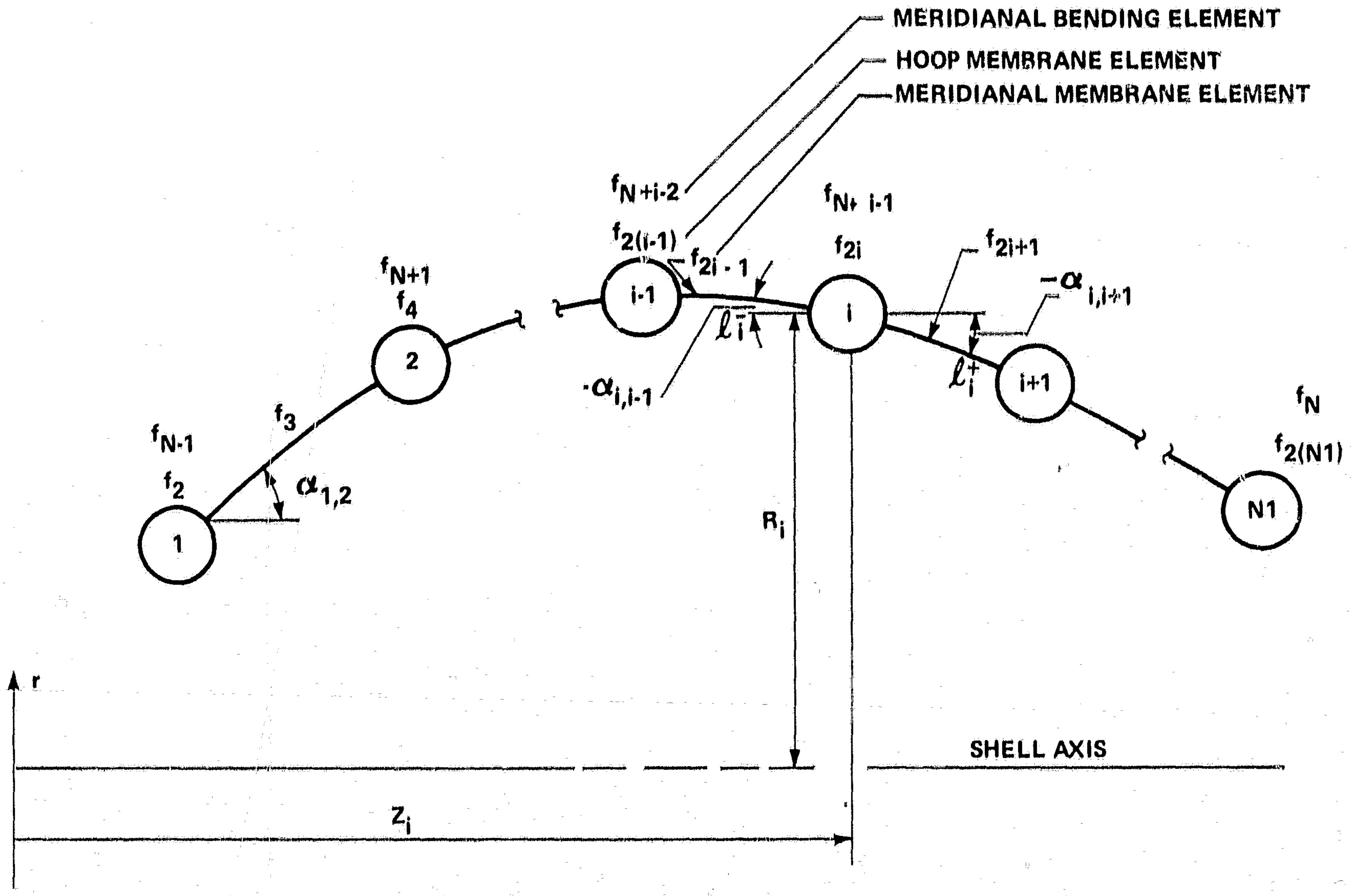
FIGURE 2 - STRESS COMPARISON

into a primary set equal to the number ($N = 2*N1+2$) of equations of equilibrium and into a redundant set of the remaining elements. The primary elements are ordered as follows (see Figure 3): (1) axial constraint, (2) hoop membrane at nodal circle 1, (3) meridianal membrane between nodal circles 1 and 2, ···(N-2) hoop membrane at nodal circle N1, (N-1) meridianal bending at nodal circle 1, and (N) meridianal bending at nodal circle N1. The ordering of the redundant elements are: 1) meridianal bending at nodal circle 2 (element N+1 of Figure 2), ···N1-2, meridianal bending at nodal circle N1-1, followed by radial constraints and terminal moment meridianal constraints if desired by the user.

The internal force in the membrane elements (including force constraints) are in units of force per length, and the internal force in the bending elements (including moment constraints) are in units of force-length per length. The sign convention for membrane is taken as extension and that of bending as compression on the outside of the shell, the sign convention of the force constraints is the positive z or r direction and that of the terminal moment constraints is in the $\vec{z} \times \vec{r}$ direction.

The force equations of equilibrium for a typical nodal circle in the axial (z) and radial (r) directions are given below.

$$
\begin{aligned}
F_{zi} = & - R_{i,i-1} \cos \alpha_{i,i-1} f_{2i-1} + R_{i,i+1} \cos \alpha_{i,i+1} f_{2i+1} \\
& + 1/2\pi \, P_{iz} + R_i f_1 + \frac{R_{i-1}}{\ell_i^-} \sin \alpha_{i,i-1} f_{N+i-2} \\
& - R_i \left(\frac{\sin \alpha_{i,i-1}}{\ell_i^-} + \frac{\sin \alpha_{i,i+1}}{\ell_i^+} \right) f_{N+i-1} \\
& + \frac{R_{i+1}}{\ell_i^+} \sin \alpha_{i,i+1} f_{N+i} = 0 \qquad (1)
\end{aligned}
$$



f DENOTES FORCE ELEMENTS-CONSTRAINTS NOT SHOWN

FIGURE 3 SHELL GEOMETRY

$$F_{ri} = - R_{i,i-1} \sin \alpha_{i,i-1} f_{2i-1} + R_{i,i+1} \sin \alpha_{i,i+1} f_{2i+1} - \ell_i f_{2i}$$

$$+ 1/2\pi \, P_{ri} + R_i f_{ir} - \frac{R_{i-1}}{\ell_i^-} \cos \alpha_{i,i-1} f_{N+i-2}$$

$$+ R_i \left(\frac{\cos \alpha_{i,i-1}}{\ell_i^-} + \frac{\cos \alpha_{i,i+1}}{\ell_i^+} \right) f_{N+i-1}$$

$$+ \frac{R_{i+1}}{\ell_i^+} \cos \alpha_{i,i+1} f_{N+i} = 0 \qquad (1a)$$

Note the force resultants F_{zi} and F_{ri} are in units of force per radian. P_{iz} and P_{ir} are virtual loads in the z and r directions, respectively, and are conjugate to the deflection degrees of freedom. The axial constraint f_1, of course, can appear in only one equation of equilibrium. The station length ℓ_i is defined as $\ell_i = 1/2(\ell_i^+ + \ell_i^-)$ and a mean radius is defined as $R_{i,i+1} = 1/2(R_i + R_{i+1})$, etc.

As previously stated moment equations of equilibrium as well as force equations of equilibrium exist at the terminal stations in the circumferential ($\vec{z} \times \vec{r}$) direction. These moment equations of equilibrium are given below.

$$M_{(\vec{z} \times \vec{r})1} = R_1 f_{N-1} + 1/2\pi \, M_1 + r_1 f_{(\vec{z} \times \vec{r})1} = 0$$

$$(1b)$$

$$M_{(\vec{z} \times \vec{r})N1} = - R_{N1} f_N + 1/2\pi \, M_{N1} + R_{N1} f_{(\vec{z} \times \vec{r})N1} = 0$$

The moment resultants $M_{(\vec{z} \times \vec{r})1}$ and $M_{(\vec{z} \times \vec{r})N1}$ are in units of moment per radian. M_1 and M_{N1} are virtual loads conjugate to the terminal rotation degrees-of-freedom. The circumferential moment constraints $(f_{\vec{z} \times \vec{r}})$ like the radial force constraints (f_{ir}) exist at the convenience at the user.

Although nodal circles at terminals 1 and N1 are permitted to have zero radii, a zero radius should be avoided by utilizing elastic constraints (see Figure 1). The elastic properties of these constraints will be considered in the next section.

Equations of equilibrium for the complete shell are written in matrix notation as follows:

$$[T_p]\{f_p\} + [T_r]\{f_r\} + 1/2\pi\ [I]\{P\} = \{0\} \tag{2}$$

where $\{f_p\}$ and $\{f_r\}$ denote the primary and redundant force elements, respectively. Letting $\{f\} = \{\frac{f_p}{f_r}\}$, the internal loads in the elements from eq. (2) are given below.

$$\{f\} = [E_r]\{f_r\} + [E_g]\{P\} \tag{3}$$

where

$$[E_r] = \left[\frac{-T_p^{-1}T_r}{I}\right]$$

and

$$[E_g] = -\ 1/2\pi\left[\frac{T_p^{-1}}{0}\right]$$

III. COMPATIBILITY

Conjugate to the internal load (f) in each force element is an internal deflection (δ). There is a linear relation between the internal deflection in an element and the internal force in the element and neighboring elements. Consider the following strain-stress relationship:

$$\begin{Bmatrix} \varepsilon^1 \\ \\ \varepsilon^2 \end{Bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu \\ -\nu & 1 \end{bmatrix} \begin{Bmatrix} \sigma^1 \\ \\ \sigma^2 \end{Bmatrix} \tag{4}$$

where σ_1 and σ_2 are stresses in the meridianal and circumferential directions, respectively. E is the modulus of elasticity and ν is Poisson's ratio. Equation (4), in terms of the membrane force elements is:

$$\begin{Bmatrix} \varepsilon^1 \\ \\ \varepsilon^2 \end{Bmatrix} = \frac{1}{Et}\begin{bmatrix} 1 & -\nu \\ -\nu & 1 \end{bmatrix} \begin{Bmatrix} f^1 \\ \\ f^2 \end{Bmatrix} \tag{5}$$

where t is the thickness of the shell. For axisymmetric loadings the circumferential curvature is zero. After integration throughout the thickness of the shell, the curvature expressions are as follows:

$$\begin{Bmatrix} \kappa^1 \\ \\ \kappa^2=0 \end{Bmatrix} = \frac{12}{Et^3}\begin{bmatrix} 1 & -\nu \\ -\nu & 1 \end{bmatrix} \begin{Bmatrix} f_b^1 \\ \\ f_b^2 \end{Bmatrix} \tag{5a}$$

where f_b^1 and f_b^2 are internal moments in the meridianal and circumferential directions, respectively. The solution of eq. (5a) yields the following expressions.

$$f_b^2 = \nu f_b^1 \tag{5b}$$

$$\kappa^1 = \frac{12(1-\nu^2)}{Et^3} f_b^1 \tag{5c}$$

Hence, the circumferential bending moment is a consequence of the meridianal bending and will not appear explicitly in the formulation; however, the meridianal curvature must be modified accordingly by the factor $(1-\nu^2)$.

The internal deflections in the force elements is obtained by integration of eqs. (5 and 5c) over the appropriate area of the shell.

The area between two stations (i and i+1) is divided into an A_i^+ and an A_{i+1}^- area as follows.

$$\begin{aligned} A_i^+ &= \pi \ell_i^+ \left(3/4\ R_i + 1/4\ R_{i+1}\right) \\ A_{i+1}^- &= \pi \ell_i^+ \left(1/4\ R_i + 3/4\ R_{i+1}\right) \end{aligned} \tag{5d}$$

The relationship between the internal deflections and loads in the force elements yields a symmetric compliance matrix [Z] for the entire shell. In matrix notation this relationship can be stated as follows.

$$\{\delta\} = [Z]\{f\} \tag{6}$$

The following is a list of compliances for the elements.

$$Z_{2i,2i} = \left(A_i^+ + A_i^-\right)/Et_i$$

$$Z_{2i+1,2i+1} = \left(\frac{A_i^+}{t_i} + \frac{A_{i+1}^-}{t_{i+1}}\right)/E$$

$$Z_{2i-1,2i} = -A_i^- \nu/Et_i$$

$$Z_{2i,2i+1} = -A_i^+ \nu/Et_i$$

interior membrane coupling

$$Z_{2,3} = -A_1^+ \nu/Et_1$$

$$Z_{N-3,N-2} = -A_{N1}^- \nu/Et_{N1}$$

terminal membrane coupling

$$Z_{1,1} = \frac{2\pi R_i}{k} \text{ if } k \text{ is supplied}$$

$$= 0 \quad \text{if } k=0$$

(Compliances for radial force constraints and circumferential moment constraints are computed in the same manner as for the compliance ($Z_{1,1}$) for the axial force constraint.)

$$Z_{N-1,N-1} = \frac{12(1-\nu^2)A_1^+}{t_1^3}$$

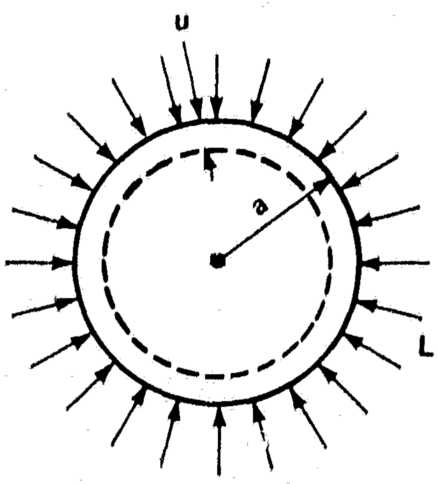
$$Z_{N,N} = \frac{12(1-\nu^2)A_{N1}^-}{t_{N1}^3}$$

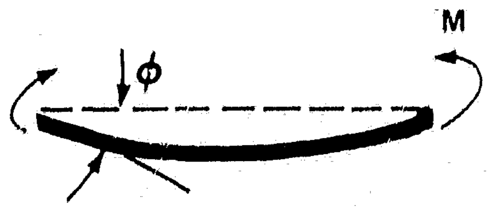
terminal meridianal bending

$$Z_{N+i,N+i} = \frac{\left(A_{i+1}^- + A_{i+1}^+\right)12(1-\nu^2)}{t_{i+1}^3}$$

interior meridianal bending

For shells without terminal cutout it is best to model the terminal stations with finite radii and to treat the resulting disc as elastic radial force and circumferential moment constraints. The spring constants can be obtained as follows.





$$u = \frac{1}{k_u} L \qquad\qquad \phi = \frac{1}{k_\phi} M$$

$$k_u = \frac{Et}{(1-\nu)a} \qquad\qquad k_\phi = \frac{Et^3 M}{12(1-\nu)a}$$

(force/length^2) (force - length/length^2)

The internal deflections in terms of the loading is obtained by substituting eq. (3) into eq. (6).

$$\{\delta\} = [Z][E_r]\{f_r\} + [Z][Eg]\{P\} \tag{7}$$

Deflections, by the method of virtual work, are obtained by multiplying the deflections by the element forces produced by virtual (dummy) loads conjugate with the sought deflections. In particular, compatibility (absence of incompatible deflections) is obtained by setting deflections $\{u_r\}$ represented by redundants equal to zero.

$$\{u_r\} = [E_r]^t\{\delta\} = \{0\} \tag{8}$$

Substituting eq. (7) into eq. (8) and setting $[H] = [E_r]^t[Z][E_r]$, solving for the redundant element forces and substituting this solution into eq. (4), one then obtains the internal forces of the assembled structure.

$$\{f\} = [\Lambda]\{P\} \tag{9}$$

where

$$[\Lambda] = ([I] - [E_r][H^{-1}][E_r]^t[Z]) \ [E_g] \quad .$$

The now compatible deflections in all of the elements is simply

$$\{\delta\} = [Z][\Lambda]\{P\} \ . \tag{10}$$

The compatibility of internal deflections having been resolved, deflections conjugate with external forces {P} can be calculated. The element forces due to the dummy loads are given by $[E_g]$ (or $[\Lambda]$), and the deflections {u} are obtained by virtual work, as given below.

$$\{u\} = [\Lambda]^t\{\delta\} = [A]\{P\} \quad , \tag{11}$$

where

$$[A] = [\Lambda]^t[Z][\Lambda] \quad .$$

[A] is commonly referred to as the deflection influence matrix.

A stiffness matrix may be obtained through the relationship

$$\{P\} = [K]\{u\} \quad , \tag{12}$$

where

$$[K] = [A]^{-1} - \begin{bmatrix} N^{al.ev}_{ull} \end{bmatrix} \quad .$$

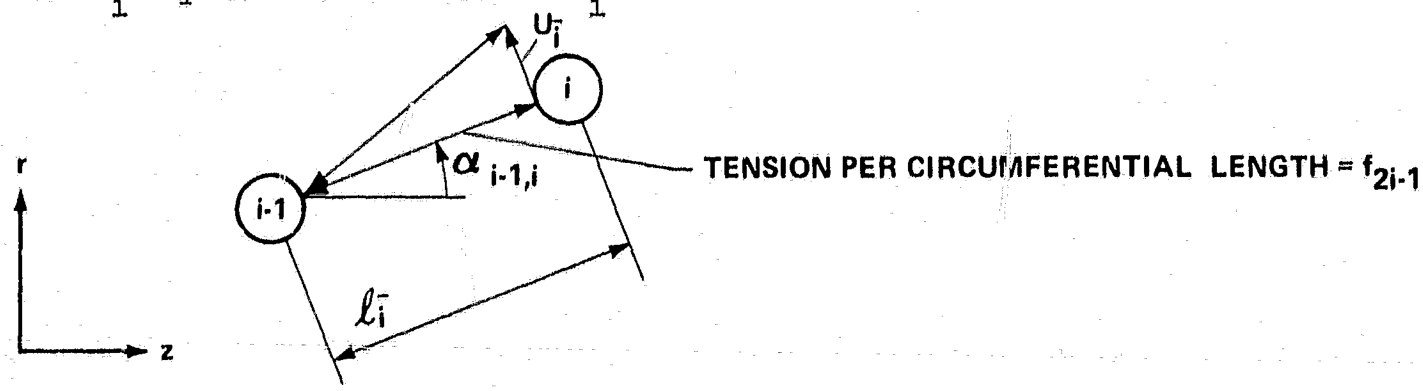
$\begin{bmatrix} N^{al.ev}_{ull} \end{bmatrix}$ is a null matrix everywhere, except at the diagonal element associated with the degree of freedom where the fictitious constraint "f_1" was connected. The non-zero diagonal element of

$\left[N^{al.ev}_{ull}\right]$ is equal to $2\pi R_i k$ (compare with $Z_{1,1}$ following eq. (6)). A reasonable value for the fictitious spring is $k \approx Et/\ell_i$. If a spring constant is not supplied then the degree-of-freedom associated with the constraint must not be supplied, or a singularity will result during the above inversion.

IV. DIFFERENTIAL STIFFNESS

Differential stiffness for axisymmetric loaded shells in this program is based on initial membrane forces (calculated relative to the axial constraint whether real or fictitious). The algorithm goes as follows.

Consider the meridianal element f_{2n-1} effects on the forces at stations i and i-1, if station i is displaced normal to ℓ_i^- by a small amount U_i^-.



If the element is in a state of tension there is a normal restoring force at station i equal to

$$2\pi R_{i,i-1} f_{2i-1} U_i^-/\ell_i^-, \tag{13}$$

and an equal and opposite force at station i-1. Likewise for the adjacent element f_{2i+1} (see Fig. 3) one obtains a normal restoring force at station i equal to

$$2\pi R_{i,i+1} f_{2i+1} \frac{U_i^+}{\ell_i^+}, \tag{13a}$$

where U_i^+ is normal to ℓ_i^+.

Let U_{zi} and U_{ri} be the axial and radial motions at station i conjugate to the external forces P_{zi} and P_{ri}. Consider now the meridianal force f_{2n+1} as well as f_{2n-1}. Now

$$U_{zi} = -\sin \alpha_{i,i-1}\, U_i \text{ for motions outward normal to } \ell_i^-$$
$$= -\sin \alpha_{i,i+1}\, U_i \text{ for motions outward normal to } \ell_i^+$$

and

$$U_{ri} = \cos \alpha_{i,i-1}\, U_i \text{ for motions outward normal to } \ell_i^-$$
$$= \cos \alpha_{i,i+1}\, U_i \text{ for motions outward normal to } \ell_i^+ \quad .$$

Now,

$$U_i^- = \left\lfloor -\sin \alpha_{i,i-1} \quad \cos \alpha_{i,i-1} \right\rfloor \begin{Bmatrix} U_{zi} \\ U_{ri} \end{Bmatrix} \tag{14}$$

for motions normal to ℓ_i^- .

Similarly

$$U_i^+ = \left\lfloor -\sin \alpha_{i,i+1} \quad \cos \alpha_{i,i+1} \right\rfloor \begin{Bmatrix} U_{zi} \\ U_{ri} \end{Bmatrix} \tag{14a}$$

for motions normal to ℓ_i^+ .

The conjugate relationships to the above equations are

$$\begin{Bmatrix} P_{zi} \\ P_{ri} \end{Bmatrix} = \begin{Bmatrix} -\sin \alpha_{i,i-1} \\ \cos \alpha_{i,i-1} \end{Bmatrix} P_i^- \tag{15a}$$

and

$$\begin{Bmatrix} P_{zi} \\ P_{ri} \end{Bmatrix} = \begin{Bmatrix} -\sin \alpha_{i,i+1} \\ \cos \alpha_{i,i+1} \end{Bmatrix} P_i^+ \qquad (15a)$$

From eqs. (13, 14, 15) and (13a, 14a, 15a) the following expression is obtained

$$\begin{Bmatrix} P_{zi} \\ P_{ri} \end{Bmatrix} = \left(\left[K_{i,i-1} \right] + \left[K_{i,i+1} \right] \right) \begin{Bmatrix} U_{zi} \\ U_{ri} \end{Bmatrix} \qquad (16)$$

where

$$\left[K_{i,i-1} \right] = \frac{2\pi \, R_{i,i-1} f_{2i-1}}{\ell_i^-} \begin{bmatrix} \sin^2 \alpha_{i,i-1} & -\sin \alpha_{i,i-1} \cos \alpha_{i,i-1} \\ -\sin \alpha_{i,i-1} \cos \alpha_{i,i-1} & \cos^2 \alpha_{i,i-1} \end{bmatrix}$$

and

$$\left[K_{i,i+1} \right] = \frac{2\pi \, R_{i,i+1} f_{2i+1}}{\ell_i^+} \begin{bmatrix} \sin^2 \alpha_{i,i+1} & -\sin \alpha_{i,i+1} \cos \alpha_{i,i+1} \\ -\sin \alpha_{i,i+1} \cos \alpha_{i,i+1} & \cos^2 \alpha_{i,i+1} \end{bmatrix}$$

By virtue of equal and opposite forces at stations i-1 and station i+1, one concludes

$$\begin{Bmatrix} P_{z(i-1)} \\ P_{r(i-1)} \end{Bmatrix} = - \left[K_{i,i-1} \right] \begin{Bmatrix} U_{zi} \\ U_{ri} \end{Bmatrix} \qquad (16a)$$

$$\begin{Bmatrix} P_{z(i+1)} \\ P_{r(i+1)} \end{Bmatrix} = - \left[K_{i,i+1} \right] \begin{Bmatrix} U_{zi} \\ U_{ri} \end{Bmatrix} \tag{16b}$$

Force contributions (eqs. (16, 16a, 16b)) from motions at each station are assembled into a differential stiffness matrix $[K_{ds}]$ implying the following relationship.

$$\{P_{ds}\} = [K_{ds}]\{u\} \tag{17}$$

Let us now expand eq. (11) to account for differential stiffness:

$$\{u\} = [A](\{P\} - [K_{ds}]\{u\}) \quad . \tag{11a}$$

The solution of the above expression is simply

$$\{u\} = [A_s]\{P\}, \tag{18}$$

where

$$[A_s] = ([I] + [A][K_{ds}])^{-1}[A]$$

is the modified deflection influence matrix. Equation (9) can be expanded in a manner similar to eq. (11).

$$\{f\} = [\Lambda](\{P\} - [K_{ds}]\{u\}) \tag{9a}$$

After substituting eq. (18) into eq. (9a) another desired relationship is obtained:

$$\{f\} = [\Lambda_s]\{P\} \tag{19}$$

where

$$[\Lambda_s] = [\Lambda](I - [K_{ds}][A_s])$$

Equation (12) must also be modified to obtain the final desired relationship.

$$\{P\} = [K_s]\{u\} \tag{20}$$

where

$$[K_s] = [K] + [K_{ds}] \quad .$$

V. LOAD VECTOR AND STRESS RECOVERY

A load vector {P} may be formed for static load analysis. The components of this vector are as follows.

$$P_{iz} = 2\pi\ R_i(PZ_i) - (A_i^- \sin \alpha_{i-1,i} + A_i^+ \sin \alpha_{i+1,i})(PS)_i$$

$$P_{ir} = 2\pi\ R_i(PR_i) + (A_i^- \cos \alpha_{i-1,i} + A_i^+ \cos \alpha_{i+1,i})(PS)_i$$

$$M_{(z \times r)1} = 2\pi\ R_1(PMF)$$

$$M_{(z \times r)N1} = 2\pi\ R_{N1}(PML)$$

$(PZ)_i$ and $(PR)_i$ are forces per circumferential length in the z and r directions, respectively, at station i. $(PS)_i$ is the outward pressure at station i. (PMF) and (PML) are moments per circumferential length at the first and last stations (terminals), respectively. Internal loads (eq. (8)) and deflections (eq. (10)) are computed for the load vector. Stresses are recovered at stations. Meridianal membrane forces must first be obtained at the stations before stress recovery can be effected. The algorithm to accomplish this is as follows.

$$f_1^{(i)} = 1/2(f_{2i+1} + f_{2i-1}) \qquad \text{interior meridianal membrane force}$$

$$\left.\begin{aligned} f_1^{(1)} &= 3/4\ f_3 - 1/4\ f_5 \\ f_1^{(N1)} &= 3/4\ f_{N-3} - 1/4\ f_{N-5} \end{aligned}\right\} \qquad \text{terminal meridianal membrane force}$$

The hoop membrane forces are directly obtained.

$$f_2^{(i)} = f_{2i}$$

The meridianal bending moments are obtained as follows.

$$f_{1b}^{(i)} = f_{N+i-1} \qquad i = 2,\cdots(N1-1) \quad \text{interior meridianal bending}$$

$$\left.\begin{aligned} f_{1b}^{(1)} &= f_{N-1} \\ f_{1b}^{(N1)} &= f_N \end{aligned}\right\} \quad \text{terminal meridianal bending}$$

Stress recovery algorithm

$$\sigma_{1m}^{(i)} = f_1^{(i)}/t_i \qquad \text{meridianal membrane}$$

$$\sigma_{2m}^{(i)} = f_2^{(i)}/t_i \qquad \text{hoop membrane}$$

$$\sigma_{1b}^{(i)} = 6\ f_{1b}^{(i)}/t_i^2 \qquad \text{meridianal bending}$$

$$\sigma_{2b}^{(i)} = \nu\ \sigma_{1b}^{(i)} \qquad \text{hoop bending}$$

$$\sigma_{1m}^{(i)} + \sigma_{1b}^{(i)} \qquad \text{inside meridianal stress}$$

$$\sigma_{1m}^{(i)} - \sigma_{1b}^{(i)} \qquad \text{outside meridianal stress}$$

$$\sigma_{2m}^{(i)} + \sigma_{2b}^{(i)} \qquad \text{inside hoop stress}$$

$$\sigma_{2m}^{(i)} - \sigma_{2b}^{(i)} \qquad \text{outside hoop stress}$$

VI. INPUT

The input is in NAMELIST format as follows

```
$NAM1*
     N1 = number of nodal circles or stations
    N31 = number of deflection degrees-of-freedom
     NN = single array of deflection degrees-of-freedom or order
          N31 keyed to the equations of equilibrium (see Note 1)
      R = single array of radii of order N1
      Z = single array of axial coordinates (see Note 2)
   EMØD = modulus of elasticity
  PØISS = Poisson's ratio
     TH = shell thickness
      E = single array of shell thickness or order N1 (only
          applicable if TH is zero or blank)
     NM = number of radial reactions
     NR = single array of order NM of radial reactions
     TI = single array of order NM of radial reaction spring
          constants
 NREACT = location of axial constraint (if zero or blank the
          constraint will be located at the first nodal circle)
  REACT = spring constant of axial reaction
  NFICT = degree-of-freedom number associated with the axial
          reaction if and only the reaction is fictitious
     NF = 0 if there is no moment constraint at station 1
          1 if there is a moment constraint at station 1
     NL = 0 if there is no moment constraint at station N1
          1 if there is a moment constraint at station N1
    ENF = spring constant of moment constraint at station 1
    ENL = spring constant of moment constraint at station N1
   PRES = outward static pressure (for load vector)
     PS = single array of order N1 of outward pressures (only
          applicable if PRES is zero or blank)
     PZ = single array of order N1 of concentrated axial forces
          per length of circumference (for load vector)
     PR = single array of order N1 of concentrated radial forces
          per length of circumference (for load vector)
    PMF = concentrated circumferential (z x r) moments per length
          of circumference at station 1 (for load vector)
    PML = concentrated circumferential (z x r) moments per length
          of circumference at station N1 (for load vector)
    N25 = 0 = no stress recovery  } prior to differential stiffness
        = 1 = stress recovery     }
```

```
  N50 = 0 = no differential stiffness
      = 1 = differential stiffness
  N75 = 0 = no stress recovery \ modified by differential stiffness
      = 1 = stress recovery    /
$END
```

2031-SK-gdn

S. Kaufman

Attachment
  Notes
  Appendix

NOTES

1. Equations of equilibrium ordering

lz, lr, 2z, ··· Nl(z), Nl(r), l(z x r), Nl(z xr)
1, 2, 3, N-3, N-2, N-1, N

2. The axial coordinates (z) of the stations must increase monotonically.

APPENDIX

TEST RUN OF SPHERICAL CAP

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@       RUN     SK,POGHES,AXSHEL,10,50

@       HDG     'SPHERICAL CAP'

@ ASG,A         SK3142.
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@       XQT     SK3142.ABS
R(1)=4.907108,R(2)=9.776495,R(3)=14.571566,R(4)=19.255726,R(5)=23.793506,
R(6)=28.15,R(7)=32.292554,R(8)=35.430716,TH=2.36,POISS=.2,EMOD=1.0E7,
Z(1)=-56.085497,Z(2)=-55.444803,Z(3)=-54.381859,Z(4)=-52.90455,
Z(5)=-51.02525,Z(6)=-48.75749,Z(7)=-46.118145,Z(8)=-43.75354,
NF=1,ENF=.279028E7,NL=1,NREACT=8,N1=8,NM=2,NR(1)=1,NR(2)=8,TI(1)=.601181E7,
N31=14,NN(1)=1,NN(2)=2,NN(3)=3,NN(4)=4,NN(5)=5,NN(6)=6,NN(7)=7,NN(8)=8,
NN(9)=9,NN(10)=10,NN(11)=11,NN(12)=12,NN(13)=13,NN(14)=14,PRES=-284.0,
PZ(1)=696.809,
N25=1,
$END
```

INTERNAL LOAD INFLUENCE MATRIX COL 1

1	-4.4920036-03	-5.7124358-02	-6.6979975-02	-1.9608763-02	-4.8649564-02	5
6	8.3790302-03	-3.2726404-02	1.7768794-02	-2.1841157-02	1.5999465-02	10
11	-1.5273244-02	9.1252363-03	-1.1746746-02	1.4876714-03	-1.0209792-02	15
16	-2.0419719-03	5.0506622-02	9.2248781-03	-2.2787596-02	-2.5983511-02	20
21	-1.7701344-02	-9.6942930-03	-3.4727859-03	2.6719674-03	6.8221873-02	25
26	-8.9460784-03	-5.0506624-02	9.2248783-03	0.0000000	0.0000000	30

INTERNAL LOAD INFLUENCE MATRIX COL 2

1	5.0384939-13	1.2795535-02	-3.3318454-03	3.9876620-03	-4.0799559-04	5
6	1.2414110-03	5.6933542-05	2.3496259-04	9.2874484-05	-7.0481157-05	10
11	6.5076101-05	-9.0070785-05	3.8376466-05	-2.6155052-05	2.9707848-05	15
16	5.9416074-06	-1.7418488-03	-1.0216731-04	7.4261186-04	8.6935705-04	20
21	5.8041129-04	2.9881752-04	1.0614863-04	-1.9865143-05	-2.1002310-02	25
26	3.5879441-05	1.7418438-03	-1.0216732-04	0.0000000	0.0000000	30

INTERNAL LOAD INFLUENCE MATRIX COL 3

1	-4.4920036-03	-3.0689652-02	-3.4725561-02	-2.8340954-02	-3.4613924-02	5
6	-6.5225316-03	-2.6984688-02	6.6177432-03	-1.9821220-02	9.8697668-03	10
11	-1.4694220-02	6.6951887-03	-1.1606707-02	1.0318846-03	-1.0138956-02	15
16	-2.0278049-03	8.1071204-03	7.6335446-03	2.0921014-02	-9.4250653-03	20
21	-1.2983263-02	-9.5929618-03	-4.7529204-03	1.1459050-03	3.7044560-02	25
26	-8.8605271-03	-8.1071205-03	7.6335447-03	0.0000000	0.0000000	30

INTERNAL LOAD INFLUENCE MATRIX COL 4

1	4.0043715-13	8.9159150-03	1.0088422-02	1.0756331-02	-2.5358185-03	5
6	4.2704564-03	-6.0028794-04	1.8405211-03	-6.8942606-05	6.2190960-04	10
11	4.9463946-05	1.2523396-04	5.7582253-05	5.4058580-06	4.9230792-05	15
16	9.8461328-06	1.8525689-03	-2.3111137-06	-3.9659075-03	-3.5423942-04	20
21	5.4870243-04	5.7091954-04	3.6588942-04	1.5030575-04	-1.0762134-02	25
26	5.9458124-05	-1.8525689-03	-2.3111137-06	0.0000000	0.0000000	30

INTERNAL LOAD INFLUENCE MATRIX COL 5

1	-4.4920035-03	-1.2436813-02	-1.4072345-02	-1.8531186-02	-1.5655825-02	5
6	-1.8898631-02	-1.9071869-02	-6.7843781-03	-1.6623132-02	1.3670844-03	10
11	-1.3575810-02	2.9709950-03	-1.1203321-02	2.8068618-04	-9.8763476-03	15
16	-1.9752825-03	-4.2072533-03	4.9490942-03	4.7173629-03	1.7405708-02	20
21	-3.4457730-03	-7.7592935-03	-5.8411130-03	-9.8412518-04	1.5012108-02	25
26	-8.5433631-03	4.2072536-03	4.9490943-03	0.0000000	0.0000000	30

INTERNAL LOAD INFLUENCE MATRIX COL 6

1	-1.7363571-11	3.9101297-03	4.4243401-03	7.8388038-03	5.7148948-03	5
6	9.5308587-03	-2.3164263-03	5.0842336-03	-6.9148976-04	2.4565159-03	10
11	-1.3814615-04	8.6858478-04	6.1753871-06	1.4722195-04	2.0569223-05	15
16	4.1138159-06	3.4249728-03	4.9562778-04	-4.2797475-04	-5.4854651-03	20
21	-9.9927001-04	4.6373275-04	7.1782181-04	6.0765161-04	-4.7198012-03	25
26	2.4842352-05	-3.4249728-03	4.9562778-04	0.0000000	0.0000000	30

INTERNAL LOAD INFLUENCE MATRIX COL 7

1	-4.4920034-03	-3.1563578-03	-3.5714403-03	-8.7786870-03	-5.5785933-03	5
6	-1.4134036-02	-7.8366631-03	-1.6183151-02	-1.2206466-02	-7.3750928-03	10
11	-1.1611842-02	-1.9160041-03	-1.0268414-02	-8.1824357-04	-9.2007648-03	15
16	-1.8401673-03	-5.3248601-03	8.9871611-04	-9.3953405-04	4.4439811-03	20
21	1.4025736-02	-1.7470789-03	-5.3919204-03	-3.3307078-03	3.8099432-03	25
26	-7.7274331-03	5.3248602-03	8.9871610-04	0.0000000	0.0000000	30

INTERNAL LOAD INFLUENCE MATRIX COL 8

1	-3.2494718-12	8.9221392-04	1.0095459-03	4.0873507-03	2.2094160-03	5
6	7.7049213-03	3.6872769-03	9.4759634-03	-2.0268098-03	5.4286994-03	10
11	-7.0207869-04	2.3643387-03	-2.4959382-04	4.7509215-04	-1.6141583-04	15
16	-3.2282817-05	3.1825496-03	1.6957916-03	1.1074964-03	-1.2666541-03	20
21	-5.9755442-03	-1.1063360-03	7.2093023-04	1.3594571-03	-1.0769639-03	25
26	-1.9494870-04	-3.1825496-03	1.6957916-03	0.0000000	0.0000000	30

INTERNAL LOAD INFLUENCE MATRIX COL 9

1	-4.4920035-03	2.4907549-04	2.8182954-04	-2.9377430-03	-9.8973127-04	5
6	-7.2712722-03	-2.7096113-03	-1.1649437-02	-4.5185781-03	-1.3470137-02	10
11	-8.6843821-03	-6.9016963-03	-8.5597698-03	-2.1500202-03	-7.8955155-03	15
16	-1.5791148-03	-3.3719344-03	-4.2157345-03	-1.8292401-03	-3.2702219-04	20
21	3.4393045-03	1.1098122-02	-1.4398773-03	-4.8899109-03	-3.0064986-04	25
26	-6.1510273-03	3.3719345-03	-4.2157346-03	0.0000000	0.0000000	30

INTERNAL LOAD INFLUENCE MATRIX COL 10

1	-3.1395630-11	-4.3058145-04	-4.8720556-04	1.5085161-03	3.0483418-04	5
6	4.3230607-03	1.4115689-03	7.4506208-03	2.6452715-03	8.7279806-03	10
11	-1.8667951-03	4.4399438-03	-9.1678320-04	1.0172039-03	-6.6878513-04	15
16	-1.3375778-04	2.1001618-03	3.7675618-03	1.2905663-03	5.8859895-04	20
21	-1.4752062-03	-6.0614904-03	-7.2005425-04	2.0573860-03	5.1974135-04	25
26	-8.0772050-04	-2.1001618-03	3.7675618-03	0.0000000	0.0000000	30

INTERNAL LOAD INFLUENCE MATRIX COL 11

1	-4.4920034-03	7.3794067-04	8.3498501-04	-4.8836130-04	3.0350613-04	5
6	-2.5648515-03	-4.9691749-04	-5.3163312-03	-1.4990572-03	-8.0130302-03	10
11	-2.5368995-03	-8.7531998-03	-6.1024451-03	-3.1998694-03	-5.9281044-03	15
16	-1.1856407-03	-1.4089721-03	-8.6445664-03	-1.1124104-03	-1.0224210-03	20
21	-9.7585641-05	2.6794767-03	8.2487901-03	-3.6411267-03	-8.9074594-04	25
26	-3.7749012-03	1.4089721-03	-8.6445665-03	0.0000000	0.0000000	30

INTERNAL LOAD INFLUENCE MATRIX COL 12

1	-3.8668580-11	-5.9366582-04	-6.7173722-04	2.3645741-04	-3.0577909-04	5
6	1.7020495-03	2.5637394-04	3.7209985-03	9.7961894-04	5.8557702-03	10
11	1.7617273-03	6.1746814-03	-2.1556035-03	1.5708824-03	-1.6580785-03	15
16	-3.3161230-04	9.7011548-04	6.0828256-03	8.1291365-04	8.2354731-04	20
21	2.7438187-04	-1.5805677-03	-5.4914870-03	1.5262939-03	7.1659642-04	25
26	-2.0025323-03	-9.7011550-04	6.0828256-03	0.0000000	0.0000000	30

INTERNAL LOAD INFLUENCE MATRIX COL 13

1	-4.4920035-03	2.5307017-04	2.8635077-04	3.7254155-05	1.8472384-04	5
6	-4.0664912-04	1.7262432-05	-1.0955590-03	-2.1782439-04	-1.9749584-03	10

11	-5.0460333-04	-2.6771064-03	-7.7925212-04	-2.5387606-03	-3.7518010-03	15
16	-7.5037554-04	-2.6934620-04	-8.3465906-03	-2.7415462-04	-3.5196477-04	20
21	-2.6983345-04	1.6626602-04	1.3307831-03	3.4423378-03	-3.0547353-04	25
26	-1.1464859-03	2.6934620-04	-8.3465908-03	0.0000000	0.0000000	30

INTERNAL LOAD INFLUENCE MATRIX COL 14

1	-5.8176903-11	-2.2355483-04	-2.5295410-04	-5.9214582-05	-1.7354071-04	5
6	2.9845566-04	-3.9363493-05	8.7429629-04	1.5435396-04	1.6452971-03	10
11	3.9880276-04	2.3360708-03	6.4605823-04	1.8646739-03	-3.0133558-03	15
16	-6.0267068-04	2.1045636-04	6.0209697-03	2.2838917-04	3.1108716-04	20
21	2.8897047-04	-5.0171212-05	-9.8309514-04	-2.7491543-03	2.6984667-04	25
26	-3.6393589-03	-2.1045636-04	6.0209697-03	0.0000000	0.0000000	30

DEFLECTION INFLUENCE MATRIX COL 1

1	3.1498629-07	-1.1347972-08	1.5915693-07	-3.4237248-09	5.2144786-08	5
6	1.0108359-08	-1.6330615-10	1.8879022-08	-1.6153032-08	1.9822646-08	10
11	-1.3120915-08	1.4078214-08	-3.8577110-09	5.0219618-09	0.0000000	15

DEFLECTION INFLUENCE MATRIX COL 2

1	-1.1347972-08	3.4935074-09	-6.1619634-09	1.7901650-09	-2.4971021-09	5
6	7.8508798-10	-6.3374279-10	1.7914125-10	5.0010028-11	-8.6453442-11	10
11	1.4816596-10	-1.1919808-10	5.0812244-11	-4.4886094-11	0.0000000	15

DEFLECTION INFLUENCE MATRIX COL 3

1	1.5915693-07	-6.1619635-09	1.3362145-07	-8.8601822-09	6.4694782-08	5
6	-2.6202216-10	1.6032068-08	9.1747896-09	-5.8417511-09	1.3393589-08	10
11	-8.7966216-09	1.1098701-08	-3.0282836-09	4.3732411-09	0.0000000	15

DEFLECTION INFLUENCE MATRIX COL 4

1	-3.4237248-09	1.7901650-09	-8.8601823-09	4.2037216-09	-6.4309573-09	5
6	2.8179724-09	-3.2442297-09	1.5522480-09	-1.1800229-09	6.2781552-10	10
11	-2.4651285-10	1.3644389-10	-3.5576823-12	-7.2622240-12	0.0000000	15

DEFLECTION INFLUENCE MATRIX COL 5

1	5.2144786-08	-2.4971022-09	6.4694782-08	-6.4309572-09	6.7079978-08	5
6	-9.4983252-09	3.3833060-08	-2.6325511-09	8.0713931-09	4.4043817-09	10
11	-2.2383010-09	6.4825860-09	-1.6429797-09	3.2604907-09	0.0000000	15

DEFLECTION INFLUENCE MATRIX COL 6

1	1.0108359-08	7.8508796-10	-2.6202226-10	2.8179723-09	-9.4983251-09	5
6	5.7098266-09	-7.8797526-09	4.3816385-09	-4.2484684-09	2.5552549-09	10
11	-1.5661844-09	1.0501330-09	-2.6246122-10	1.9656251-10	0.0000000	15

DEFLECTION INFLUENCE MATRIX COL 7

1	-1.6330595-10	-6.3374279-10	1.6032068-08	-3.2442296-09	3.3833060-08	5
6	-7.8797525-09	4.2112643-08	-1.1536458-08	2.2268958-08	-5.5358258-09	10
11	6.2196551-09	3.1810694-10	4.2012832-10	1.5456029-09	0.0000000	15

DEFLECTION INFLUENCE MATRIX COL 8

1	1.8879022-08	1.7914126-10	9.1747896-09	1.5522480-09	-2.6325513-09	5
6	4.3816385-09	-1.1536458-08	7.6168355-09	-8.8979445-09	5.7366195-09	10
11	-4.1659842-09	2.9288683-09	-8.7554274-10	7.0235834-10	0.0000000	15

DEFLECTION INFLUENCE MATRIX COL 9

1	-1.6153032-08	5.0010014-11	-5.8417511-09	-1.1800229-09	8.0713930-09	5
6	-4.2484684-09	2.2268959-08	-8.8979445-09	2.9199783-08	-1.2218158-08	10
11	1.4572827-08	-6.1701475-09	2.9814009-09	-6.7797378-10	0.0000000	15

DEFLECTION INFLUENCE MATRIX COL 10

1	1.9822645-08	-3.6453439-11	1.3393589-08	6.2781552-10	4.4043815-09	5
6	2.5552548-09	-5.5358259-09	5.7366194-09	-1.2218158-08	8.7339905-09	10
11	-7.6596279-09	5.6183555-09	-1.9151381-09	1.6003400-09	0.0000000	15

DEFLECTION INFLUENCE MATRIX COL 11

1	-1.3120915-08	1.4816596-10	-8.7966216-09	-2.4651285-10	-2.2383011-09	5
6	-1.5661844-09	6.2196553-09	-4.1659841-09	1.4572826-08	-7.6596276-09	10
11	1.7501570-08	-9.3852846-09	5.1181734-09	-2.7111647-09	0.0000000	15

DEFLECTION INFLUENCE MATRIX COL 12

1	1.4078213-08	-1.1919807-10	1.1098701-08	1.3644388-10	6.4825860-09	5
6	1.0501379-09	3.1810696-10	2.9288683-09	-6.1701476-09	5.6183554-09	10
11	-9.3852847-09	7.4227484-09	-3.0359814-09	2.6581883-09	0.0000000	15

DEFLECTION INFLUENCE MATRIX COL 13

1	-3.8577109-09	5.0812244-11	-3.0282836-09	-3.5576858-12	-1.6429798-09	5
6	-2.6246123-10	4.2012838-10	-8.7554276-10	2.9814008-09	-1.9151381-09	10
11	5.1181734-09	-3.0359814-09	4.7536037-09	-2.8674082-09	0.0000000	15

DEFLECTION INFLUENCE MATRIX COL 14

1	5.0219616-09	-4.4886094-11	4.3732411-09	-7.2622214-12	3.2604907-09	5
6	1.9656251-10	1.5456029-09	7.0235835-10	-6.7797378-10	1.6003400-09	10
11	-2.7111647-09	2.6581883-09	-2.8674082-09	2.8139041-09	0.0000000	15

INTERNAL FORCE VECTOR

1	-4.5954085+03	-7.3218489+03	-8.3350694+03	-8.4769399+03	-8.3867034+03	5
6	-8.1771991+03	-8.3206923+03	-7.5301821+03	-8.1447067+03	-6.2704870+03	10
11	-7.8245769+03	-4.3156137+03	-7.3510501+03	-2.1355329+03	-6.8420235+03	15
16	-1.3684203+03	1.0292451+02	-4.5985453+03	2.1827332+02	5.0934757+02	20
21	8.4214630+02	9.6965469+02	4.3589447+02	-1.4610803+03	8.8222744+03	25
26	-4.8007643+03	-1.0292452+02	-4.5985454+03	0.0000000	0.0000000	30

DEFLECTION VECTOR

1	2.2734585-02	-1.4674901-03	2.2237474-02	-2.8159895-03	2.1175073-02	5
6	-4.0138559-03	1.8984241-02	-4.7968798-03	1.5088823-02	-4.7088940-03	10
11	9.3991181-03	-3.3364710-03	3.0804202-03	-9.6741705-04	9.7006048+04	15

MEMBRANE STRESS VECTOR MERIDIONAL STATION

1	-3.5208697+03	-3.5427485+03	-3.5397025+03	-3.4884320+03	-3.3833228+03	5
6	-3.2151753+03	-3.0070071+03	-2.7913179+03	0.0000000	0.0000000	10

MEMBRANE STRESS VECTOR HOOP STATION

1	-3.1024784+03	-3.5919237+03	-3.4649149+03	-3.1907551+03	-2.6569860+03	5
6	-1.8286499+03	-9.0488683+02	-5.7983913+02	0.0000000	0.0000000	10

BENDING STRESS VECTOR MERIDIONAL STATION

1	1.1087818+02	2.3514075+02	5.4870825+02	9.0722455+02	1.0445864+03	5

6	4.6957894+02	-1.5955332+03	-4.9539055+03	0.0000000	0.0000000	10

BENDING STRESS VECTOR HOOP

1	2.2175635+01	4.7028149+01	1.0974165+02	1.8144491+02	2.0891728+02	5
6	9.3915786+01	-3.1910665+02	-9.9078108+02	0.0000000	0.0000000	10

STRESS VECTOR MERIDIONAL INSIDE SURFACE

1	-3.4099915+03	-3.3076078+03	-2.9909943+03	-2.5812075+03	-2.3387364+03	5
6	-2.7455963+03	-4.6025403+03	-7.7452233+03	0.0000000	0.0000000	10

STRESS VECTOR MERIDIONAL OUTSIDE SURFACE

1	-3.6317479+03	-3.7778892+03	-4.0884107+03	-4.3956565+03	-4.4279092+03	5
6	-3.6847541+03	-1.4114739+03	2.1625876+03	0.0000000	0.0000000	10

STRESS VECTOR HOOP INSIDE SURFACE

1	-3.0803028+03	-3.5448956+03	-3.3551733+03	-3.0093102+03	-2.4480688+03	5
6	-1.7347341+03	-1.2239935+03	-1.5706202+03	0.0000000	0.0000000	10

STRESS VECTOR HOOP OUTSIDE SURFACE

1	-3.1246540+03	-3.6389519+03	-3.5746565+03	-3.3722000+03	-2.8659033+03	5
6	-1.9225657+03	-5.8578018+02	4.1094195+02	0.0000000	0.0000000	10

STIFFNESS MATRIX COL 1

1	1.1701765+07	2.6447287+07	-1.7347627+07	-2.8536970+07	5.6473091+06	5
6	-1.6688614+06	-1.4258343+03	-4.6693011+03	-2.1204595+01	-5.2528080+01	10
11	-8.7840513-01	-2.1684749+00	1.0642662+00	2.2229575+00	0.0000000	15

STIFFNESS MATRIX COL 2

1	2.6447286+07	4.3968803+08	-2.5083578+07	-2.1885687+08	-1.3534480+06	5
6	-2.6554338+06	-1.0112054+04	-3.3115025+04	-1.4399268+02	-3.5788134+02	10
11	-5.6824894+00	-1.1938437+01	1.6973243-01	4.2689046-02	0.0000000	15

STIFFNESS MATRIX COL 3

1	-1.7347627+07	-2.5083578+07	5.4624071+07	9.9573013+07	-4.5198968+07	5
6	-7.4004797+07	7.9268477+06	-3.5424682+06	-4.2639659+03	-1.0578666+04	10
11	-5.7815377+01	-1.1077296+02	-7.0081667+00	-1.6346267+01	0.0000000	15

STIFFNESS MATRIX COL 4

1	-2.8536970+07	-2.1885687+08	9.9573011+07	6.4616942+08	-6.7952363+07	5
6	-3.4829998+08	-3.0651364+06	-3.4983930+06	-1.8305393+04	-4.5397067+04	10
11	-2.2904902+02	-4.4239733+02	-1.3496536+01	-3.6015311+01	0.0000000	15

STIFFNESS MATRIX COL 5

1	5.6473105+06	-1.3534413+06	-4.5198972+07	-6.7952377+07	1.1326061+08	5
6	2.1117956+08	-8.3295152+07	-1.3848264+08	9.5955434+06	-5.9421075+06	10
11	-9.2378683+03	-1.8159736+04	-1.1034986+02	-1.7253574+02	0.0000000	15

STIFFNESS MATRIX COL 6

1	-1.6688593+06	-2.6554178+06	-7.4004801+07	-3.4830000+08	2.1117955+08	5
6	8.7388929+08	-1.3015796+08	-4.6587566+08	-5.3193641+06	-3.8118956+06	10
11	-2.8232698+04	-5.5509977+04	-3.6116692+02	-5.6197904+02	0.0000000	15

STIFFNESS MATRIX COL 7

1	-1.4286903+03	-1.0124561+04	7.9268576+06	-3.0651095+06	-8.3295159+07	5
6	-1.3015798+08	2.0431774+08	3.5565445+08	-1.3944986+08	-2.1621121+08	10
11	1.0518689+07	-8.6871237+06	-1.6621841+04	-2.6641487+04	0.0000000	15

STIFFNESS MATRIX COL 8

1	-4.6753453+03	-3.3148286+04	-3.5424496+06	-3.4983368+06	-1.3848265+08	5
6	-4.6587569+08	3.5565446+08	1.0741019+09	-2.0566533+08	-5.6069668+08	10
11	-7.9198650+06	-3.4082486+06	-3.8986312+04	-6.2484433+04	0.0000000	15

STIFFNESS MATRIX COL 9

1	-1.9768102+01	-1.3863524+02	-4.2728396+03	-1.8328970+04	9.5955471+06	5
6	-5.3193644+06	-1.3944993+08	-2.0566524+08	3.3573778+08	5.2161013+08	10
11	-2.1645562+08	-3.0142877+08	1.0606176+07	-1.1567374+07	0.0000000	15

STIFFNESS MATRIX COL 10

1	-4.8370016+01	-3.4315249+02	-1.0596509+04	-4.5450192+04	-5.9421012+06	5
6	-3.8118992+06	-2.1621117+08	-5.6069651+08	5.2161015+08	1.2291009+09	10
11	-2.8872914+08	-6.2911422+08	-1.0661318+07	-2.2188871+06	0.0000000	15

STIFFNESS MATRIX COL 11

1	1.0340276+00	-9.2383999-01	-6.0662897+01	-2.3233618+02	-9.2230670+03	5
6	-2.8188027+04	1.0518638+07	-7.9199965+06	-2.1645556+08	-2.8872902+08	10
11	5.1336465+08	6.9561256+08	-3.2217196+08	-3.8252806+08	0.0000000	15

STIFFNESS MATRIX COL 12

1	7.3987655-01	-4.6884337+00	-1.1558793+02	-4.4445610+02	-1.8137216+04	5
6	-5.5440219+04	-8.6872041+06	-3.4084614+06	-3.0142870+08	-6.2911402+08	10
11	6.9561256+08	1.3317490+09	-3.6814647+08	-6.7216875+08	0.0000000	15

STIFFNESS MATRIX COL 13

1	-1.2869461+00	-3.8119177+00	5.8448380-01	1.2240267+00	-1.2557524+02	5
6	-4.0166713+02	-1.6596394+04	-3.8918971+04	1.0606156+07	-1.0661380+07	10
11	-3.2217196+08	-3.6814646+08	8.7957128+08	9.4229839+08	0.0000000	15

STIFFNESS MATRIX COL 14

1	-1.5090729+00	-5.1607409+00	-2.4230932+00	-1.0368254+01	-1.9784317+02	5
6	-6.2639124+02	-2.6604556+04	-6.2390933+04	-1.1567410+07	-2.2189926+06	10
11	-3.8252804+08	-6.7216872+08	9.4229837+08	1.5805098+09	0.0000000	15

@ FIN

RUNID: SK   ACCOUNT: POGUES   PROJECT: AXSHEL

TIME: 00:00:02.563   IN: 16   OUT: 0   PAGES: 10

INITIATION TIME: 03:28:49-JUL 24,1969

TERMINATION TIME: 03:29:16-JUL 24,1969

CORE-SECONDS: 26

IO COUNT: 33

CHARGE: 0.542