LESSON 10:
BLOCKING MODELS AND LOSS ESTIMATES

Objective:

The objective here is to learn about blocking modes and loss estimates

Introduction:

We have already studied that telecommunication switching systems can be classified as loss systems and delay systems. This classification is based on the way in which overflow traffic is handled. The behaviour of loss systems is studied by using blocking models. But the behaviour of delay systems is studied by using queuing models. In this section, we study the analysis of loss systems.

Blocking Models And Loss Estimate

We have three aspects while dealing with the analysis of the telecommunication switching systems:

1. Modelling of the Telecommunication Switching System
2. Traffic Arrival Model
3. Service Time Distribution.

Modelling of the Telecommunication Switching System:

First of all, we model the system as a birth-death (B-D) process, arrival as a Poisson process and the holding time as an exponential or a constant time distribution.

Traffic Arrival Model

In loss systems, the overflow traffic is rejected. In other words, we can say that the over flow traffic experiences blocking from the network.

There are 3 ways in which overflow traffic, rejected in loss systems, may be handled:

The traffic rejected by one set of resources may be cleared by another set of resources in telecommunication switching network.

i. The traffic rejected by one set of resources may be cleared by another set of resources in telecommunication switching network.
ii. The traffic may return to the same resource after sometime.
iii. The traffic may he held by the resource as if being serviced but actually serviced only after the resource become available.

Case (iii) above is not the one where a call is queued and serviced. Here, a call is accepted and the customer is allowed to proceed with his information exchange process. No resources are allocated immediately although the call has been accepted. Some part of the initial information from the subscriber may be lost.

Now we are going to consider three models of loss systems corresponding to the above three cases:

a. Lost Calls Cleared (LCC)
b. Lost Calls Returned (LCR)
c. Lost calls Held (LCH)

Lost Calls Cleared (LCC) may be discussed in parts such as

a. Lost Calls Cleared (LCC) system with infinite resources
b. Lost Calls Cleared (LCC) system with finite subscribers.

1) Lost Calls Cleared (LCC) Systems with Infinite Sources:

We first discuss analysis of an LCC system assuming infinite number subscribers. The assumption permits us to use Poisson arrival model for the traffic. The arrival rate is independent of the number of subscribes already busy. The arrival rate remains constant irrespective of the state of the system. Lost Calls Cleared (LCC) Model is most suitable for the study of the behaviour of trunk transmission systems. In usual way, there are many trunk groups emanating from a switching office and terminating on adjacent switching offices. It is possible to divert the traffic via other switching offices using different trunk groups whenever a direct trunk group between two switching offices is busy. In this way, the blocked calls in one trunk group are cleared via other trunk groups. In the context of' subscriber calls, the LCC model assumes that a subscriber on hearing the engaged tone, hands up and waits for some length of time before- reattempting.

Subscriber cannot reattempt immediately or within short time. Such type of calls are considered to have been c1eared from the system. Reattempts of calls are treated as new calls. The LCC model is used as a standard for the design and analysis of telecommunication networks in India, Europe and other countries that adopt European practices.

The LCC model was first studied by A.K. Erlang in 1917. The main purpose of the analysis is to estimate the blocking probability P_B and the grade of service (GOS). We can express the offered traffic A for Poisson arrival process as,

$A = R_p\, t_h$ ………………….. Eq.1 Where,

Rp = Average Poisson call arrival rate

We already know that R represents the average call arrival rate irrespective of the arrival distribution but Rp represents the Poisson arrival rate here.

When all the servers (or links or trunks) in the system are busy, any traffic generated by the Poisson process is generated by the system. Since in loss system, the overflow traffic is lost, as far as, the network is concerned; there is different arrival rate, which is called effective arrival rate. Mean effective arrival rate is denoted by Ro Effective arrival rate in state i is denoted by R_i. The system is said to be in state j when j servers are busy. When all the servers in the system are not busy, the entire incoming traffic is carried by the network. When all the servers are busy, no traffic is accepted by the network. Such traffic on the network is known as Erlang traffic or pure chance traffic of type-l. In this case, we have

$R_i = R_p$ for $N_L > i$ ³ 0

where,

N_L = Number of servers or links in the system.

The mean effective traffic rate R_o is determined as

$$R_0 = \sum_{i=0}^{N_L - 1} R_p P_i \qquad \text{……………….. Eq.2}$$

where,

P_i = Probability that the system is in state i

The system will be in any one of 0,1,2,……N_L states. Therefore, we have,

$P_0 + P_1 + P_2 + \dots + P_{NL} = 1$ …………. Eq.3

Expanding Eq.2, we get

$Rp = Rp (P_0 + P_1 + P_2 + \dots + P_{NL-1})$ ………..Eq.4

From Eqs.2 and 4; we get,

$R_0 = R_p (1- P_{NL})$ …………… Eq.5

The-mean traffic carried by the network is given by,

Ao = Mean effective traffic rate * t_h

$= R_0 t_h$

$= R_p (1- P_{NL}) t_h$ [From Eq.5]

or $A_0 = R_p (1- P_{NL}) t_h$ …………. Eq.6

Eq.6 can be written as,

$A_0 = R_p t_h (1- P_{NL})$ …………. Eq.7

We know that the offered traffic A for Poisson arrival process is given as,

$A = R_p t_h$ …………………Eq.8

Substituting Eq.8 in Eq.7, we get

$A_0 = A (1- P_{NL})$

Or $P_{NL} = \dfrac{A - A_0}{A}$ ……….. Eq.9

The blocking probability P_B is the same as the probability that all the servers are busy i.e. P_{NL}. Therefore, from following equations

$$GOS = \frac{A - A_0}{A}$$

And $P_B + P_{NL} = \dfrac{A - A_0}{A}$, we conclude that

$GOS = P_B = P_{NL}$

For LCC model where the traffic arrival is characterized by Poisson process.

Now we shall calculate the blocking probability P_B, for calculating P_B, we perform the steady state analysis of the B-D process characterizing the LCC model. We have already discussed the call arrival process. Now we shall discuss call termination process. A constant death-rate is not suitable here. The call termination rate should be dependent on the number of busy servers in the system, i.e. state of the system. If a large number of servers are busy, more cells are likely to terminate in a given time and call termination rate will be higher.

Hence, the call termination rate is considered directly proportional to the number of busy servers.

It is given by,

r_k = call termination rate in state k

$= k\, r_m$ for $N_L > i ³ 0$ ………….. Eq.10

where,

r_m = mean call termination rate

$= 1 / t_h$

Steady state equations of a B-,D process are given by,

$P_{k-1} R_{k-1} + P_{k+1} r_{k+1} - (R_k + r_k) P_k = 0$, for k ³ 1 ………… Eq.11

and $P_1 r_1 - R_0 P_0 = 0$ for k = 0 …………Eq.12

Substituting the values of birth and death rates in Eqs.11 & 12, we get

$P_{k-1} R + P_{k+1} (k+1) r_m - (R + r_k) P_k = 0$ ………. Eq.13

$P_1 r_m - R P_0 = 0$ ……….. Eq.14

Now,

$A = R t_h$

Substituting in eq.13, we get

$$P_{k-1} R + P_{k+1}(k+1)\frac{1}{t_h} - \left(R + \frac{k}{t_h}\right)P_k = 0$$

or $$P_{k-1} Rt_h + P_{k+1}(k+1) - Rt_h P_k - kP_k = 0$$

or $$P_{k-1} A + P_{k+1}(k+1) - AP_k - kP_k = 0$$

or $$P_{k+1} = \frac{AP_k + kP_k - AP_{k-1}}{k+1}, \text{ for } k > 0 \qquad \textbf{Eq.15}$$

Using Eqn. $A = Rt_h$ in Eqn. **14** we get

$$P_1 r_m = RP_0$$

or $$P_1 \frac{1}{t_h} = RP_0, \text{ Since } r_m = \frac{1}{t_h}$$

or $$P_1 = Rt_h P_0, \text{ since } A = Rt_h$$

or $$P_1 = AP_0, \quad \text{for} \quad k = 0 \qquad \ldots\ldots \textbf{Eq.16}$$

For $k = 1$, we can get from Eqn. 16

$$P_2 = \frac{AP_1 + P_1 - AP_0}{2} \qquad \ldots\ldots \text{Eq.17}$$

Substituting Eqn. 16 in Eqn. 17, we get

$$P_2 = \frac{A^2 P_0}{2} \qquad \ldots\ldots \text{Eq.18}$$

For $k = 2$, we can get from Eqn. 15

$$P_3 = \frac{AP_2 + 2P_2 - AP_1}{3} \qquad \text{Eq.19}$$

Substituting Eqns. 16 and 18 in Eqn. 19 we

$$P_3 = \frac{A^3 P_0}{6} = \frac{A^3 P_0}{\angle 3} \qquad \text{Eq.20}$$

On generalisation, we get

$$P_j = \frac{A^j P_0}{\angle j} \qquad \text{Eq.21}$$

From eqns. $P_0 + P_1 + P_2 + \ldots P_{N_L}$ and $P_j = \frac{A^j P_0}{\angle j}$

$$P_0 + \frac{AP_0}{\angle 1} + \frac{A^2 P_0}{\angle 2} + \frac{A^3 P_0}{\angle 3} + \ldots + \frac{A^{N_L} P_0}{\angle N_L} = 1$$

Therefore,

$$P_0 = \frac{1}{1 + \frac{A}{\angle 1} + \frac{A^2}{\angle 2} + \frac{A^3}{\angle 3} + \ldots + \frac{A^{N_L}}{\angle N_L}} \qquad \text{Eq.22}$$

This is the famous Erlang B formula or loss formula.

The quantity P_{NL} is the probability that is servers or links are busy in the system and hence is the blocking probability P_B of the system. We have already seen that for the Erlang traffic (which occurs in the LCC model) the GOS and P_B values are equal. Therefore, the values of GOS is given by Erlang B formula. CCITT has adopted Erlang B formula as standard for estimating system.

When N_L is large and A is small then A / N_L will be very small. Therefore, denominator Eq.22 reduces to e^{-A}. Eq.21 then becomes,

For $j = N_L$, Eq.21 can be written as,

$$P_{NL} = \frac{A^{N_L} P_0}{\angle N_L} \qquad \ldots\ldots\ldots\ldots\text{Eq.23}$$

Substituting Eq.22 in Eq.23, we get

$$P_{NL} = \frac{\dfrac{A^{N_L} P_0}{\angle N_L}}{1 + (A / \angle 1) + (A^2 / \angle 2) + \ldots\ldots (A^N / \angle N_L)} \qquad \ldots\ldots\text{Eq.24}$$

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When N_L is large and A is small then A / N_L will be very small. Therefore, denominator Eq.22 reduces to e^{-A}. Eq.21 then becomes,

$$P_j = \frac{A_j}{\angle j} e^{-A} \qquad \ldots\ldots\text{Eq.25}$$

This equation is same as Poisson Equation. In this case traffic is Poisson. Taking the limit $\longrightarrow N_L \infty$, we have

$$\lim_{N_L \to \infty} \frac{ANL \; e\text{-}A}{\angle N_L} \rightarrow 0$$

or $$\bar{P}_B \rightarrow 0 \qquad \ldots\ldots\ldots \text{Eq.26}$$

Thus, the blocking probability P_B tends to zero with Poisson traffic.

Lost Calls Cleared (LCC) System with Finite subscribers

Erlang loss formula or Erlang B formula was derived under a fundamental assumption that call arrivals are independent of the number of active callers. This assumption is true only when the number of sources is much larger than the number of servers or links. This may not always be the case in practice. For example, in a three-stage space division network, the number of subscribers connected to each input matrix is comparable to the number of servers (alternative paths) available in the network. In such cases, the arrival rate to the system is dependent on the number of subscribers who are not occupied, as the busy subscribers do not generate new calls. Such type of traffic is called Engest traffic or pure chance traffic of type-2.

Now we derive expressions that govern blocking probabilities in such cases.

The blocking probabilities in finite source systems are always less than those for infinite source systems since the arrival rate decreases as the number of busy sources increased.

Let us assume following quantities and their symbols:

R_s = Arrival rate per subscriber

K = Number of busy subscribers

N_s = Total number of subscribers

N_L = Number of servers or links

The offered traffic or arrival rate when the system is in state k is given by

$R_k = (N_s - k) R_s$. for $K \leq 0 \leq N_L$

The mean offered traffic rate is given by

$$R = \sum_{k=0}^{N_L} (N_s - k) R_s P_k$$

$$= \sum_{k=0}^{N_L} N_s R_s P_k - \sum_{k=0}^{N_L} k R_s P_k$$

or $$= N_s R_s \sum_{k=0}^{N_L} P_k - R_s \sum_{k=0}^{N_L} k P_k$$

$$\text{Since } \sum_{k=0}^{N_L} P_k = 1$$

$$R = N_s R_s - R_s \sum_{k=0}^{N_L} k P_k$$

The carried traffic in a network is the average number of calls accepted during the mean service time period. The carried traffic is the same as the average number of busy servers or links at any given time. Hence, it is given by

$R = R_s (N_s - A_o)$ …………..Eq.28

The offered traffic is

$A = R t_h = R_s (N_s - A_0) t_h = R_s t_h (N_s - A_0)$ …………Eq.29

traffic rate is $(N_s - N_L) R_s$, but all the arrivals are rejected. Therefore,

Lost traffic = $A - A_0 = (N_s - N_L) R_s P_{NL} t_h$ ………………Eq.30

The GOS now works out to be,

$$GOS = \frac{Ns - NL}{N_s - A_0} P_{NL} \quad \text{…………Eq.31}$$

Thus, we can conclude that for Engest traffic, the blocking probability P_B and GOS are not same. Therefore, call congestion and time congestion values differ.

Now we shall calculate the blocking probability. For calculating P_B we analyse the steady state of B-D process which characterize this model. The call termination process is the same as in the case of LCC model infinite sources, Call termination rate is given by,

$$r_k = k\, r_m, \quad \text{for } 0 \leq k \leq N_L$$

where,

r_k = Call termination rate in state k

r_m = Mean call termination rate = $\frac{1}{t_h}$

Putting the values of birth and death rates in Eqns. 11 and 12 we have

$$P_{k-1} R_S (N_S - k + 1) + P_{k+1} r(k+1) - [R_S (N_S - k) + kr] P_k = 0$$

On rearranging the terms, we obtain

$$P_{k+1} = \frac{\left[\left\{\frac{R_S}{\mu}(N_S - k) + k\right\}\right] P_k - \frac{R_S}{r}(N_S - k + 1) P_{k-1}}{k+1} \quad \text{..Eq.32}$$

Substituting $r = \frac{R_S}{r}$, we get

$$P_{k+1} = \frac{[\{\rho(N_S - k) + k\} P_k - \rho(N_S - k + 1) P_{k-1}]}{k+1} \quad \text{....Eq.33}$$

For $k = 0$, we have

$$P_1 = \frac{R_S}{r} N_S P_0 \quad \text{or} \quad P_1 = \rho N_S P_0 \quad \text{..... Eq.34}$$

For $k = 1$,

$$P_2 = \frac{[\{\rho(N_S -) + 1\} P_1 - \rho(N_S - 1 + 1) P_0]}{2}$$

$$= \frac{[\{\rho(N_S - 1) + 1\} P_1 - \rho N_S P_0]}{2}$$

$$= \frac{[\{\rho(N_S - 1) + 1\} \rho N_S P_0 - \rho N_S P_0]}{2}$$

or $$P_2 = \frac{\rho^2 N_S (N_S - 1) P_0}{2} \quad \text{..... Eq.35}$$

For $k = 2$, we have

$$P_3 = \frac{\rho^3 N_S (N_S - 1)(N - 2) P_0}{3 \times 2} \quad \text{..... Eq.36}$$

On generalisation of above, we get

$$P_j = \rho^j \binom{N_S}{j} P_0 \quad \text{..... Eq.37}$$

where $\binom{N_S}{j}$ is the Binomial Coefficient.

This Binomial Coefficient is defined as

$$\binom{N_S}{j} = \frac{\angle N_S}{\angle j \ \angle (N_S - j)}$$

Already, we know that

$$P_0 + P_1 + P_2 + \dots + P_{N_L} = 1 \quad \text{..... Eq.38}$$

From Eqns. 37 and 38, we get

$$P_0 + \rho^1 \binom{N_S}{1} P_0 + \rho^2 \binom{N_S}{2} P_0 + \rho^3 \binom{N_S}{3} + \dots + \rho^{N_L} \binom{N_S}{N_L} P_0 = 1$$

Therefore,

$$P_0 = \frac{1}{1 + \rho \binom{N_S}{1} + \rho^2 \binom{N_S}{2} + \dots\dots + \rho^{N_L} \binom{N_S}{N_L}}$$

or $$P_0 = \frac{1}{\sum_{k=0}^{N_L} \rho^k \binom{N}{k}} \quad \text{..... Eq.39}$$

The blocking probability P_B is given by

$$P_B = P_{N_L} = \frac{\rho^{N_L} \binom{N_S}{N_L}}{\sum_{k=0}^{N_L} \rho^k \binom{N_S}{k}} \quad \text{..... Eq.40}$$

From Eqns. $GOS = \frac{N_S - N_L}{N_S - A_0} P_{N_L}$ and Eqn. 40, we get

$$\text{GOS} = \frac{N_S - N_L}{N_S - A_0} P_{N_L} = \left(\frac{N_S - N_L}{N_S - A_0}\right) \frac{\rho^{N_L}\binom{N_S}{N_L}}{\sum_{k=0}^{N_L} \rho^k \binom{N_S}{k}}$$

$$= \left(\frac{N_S - N_L}{N_S - A_0}\right) \rho^{N_L} \frac{\angle N_S}{\angle N_L \; \angle N_S - N_L} \cdot \frac{1}{\sum_{k=0}^{N_L} \rho^k \binom{N_S}{k}}$$

$$= \frac{(N_S - N_L)}{(N_S - A_0)} \rho^{N_L} \frac{N_S \; \angle N_S}{\angle N_L \cdot (N_S - N_L) \; \angle N_S - N_L - 1} \cdot \frac{1}{\sum_{k=0}^{N_L} \rho^k \binom{N_S}{k}}$$

$$= \frac{N_S}{(N_S - A_0)} \rho^{N_L} \left\{ \frac{\angle N_S - 1}{\angle N_L \; \angle N_S - N_L - 1} \right\} \cdot \frac{1}{\sum_{k=0}^{N_L} \rho^k \binom{N_S}{k}}$$

or $$\text{GOS} = \frac{N_S}{(N_S - A_0)} \rho^{N_L} \binom{N_S - 1}{N_L} \cdot \frac{1}{\sum_{k=0}^{N_L} \rho^k \binom{N_S}{k}} \qquad \text{......Eq.41}$$

From Eqns. 27, 28 and 40 ,

We get

$$N_s - A_0 = \frac{N_S \sum_{k=0}^{N_L} \rho^k \binom{N_S}{k} - \sum_{j=0}^{N_L} j\rho^j \binom{N_S}{j}}{\sum_{k=0}^{N_L} \rho^k \binom{N_S}{k}}$$

We can combine both terms in the numerator because limits of summation are same.

$$N_S - A_0 = \frac{\sum_{k=0}^{N_L} \rho^k \binom{N_S}{k} (N_S - k)}{\sum_{k=0}^{N_L} \rho^k \binom{N_S}{k}}$$

On simplification of above equation, we get

$$N_S - A_0 = \frac{N_S \sum_{k=0}^{N_L} \rho^k \binom{N_S - 1}{k}}{\sum_{k=0}^{N_L} \rho^k \binom{N_S}{k}} \qquad \text{......Eq.42}$$

Substituting the value of $(N_s - A_0)$ from Eqn. 42 in Eqn. 41 , we get

$$\text{GOS} = \frac{\rho^{N_L}\binom{N_S - 1}{N_L}}{\sum_{k=0}^{N_L} \rho^k \binom{N_S - 1}{k}} \qquad \text{......Eq.43}$$

An important case of the LCC model with finite source occurs when $N_L \le N_S$.

In this case, there is no blocking. Blocking probability of state j is given by

$$P_j = \binom{N_S}{j} \frac{\rho^j}{\sum_{k=0}^{N_S} \rho^k \binom{N_S}{k}} = \binom{N_S}{j} \frac{\rho^j}{(1+\rho)^{N_S}} \qquad \text{Eq.44}$$

In this case, traffic is Bernaulli traffic.

If we define, $\alpha = \frac{\rho}{1+\rho}$, then

$$\rho = \alpha(1 + \rho)$$

or $\rho = \alpha + \rho\alpha$ or $\rho = \alpha(1 + \rho)$ Eq.45

or $\alpha = \frac{-\rho}{1+\rho}$ or $1 - \alpha = \frac{1}{1+\rho}$ Eq.46

Substituting Eqns. 45 and 46 in Eqn. 44 , we get

$$P_j = \binom{N_S}{j} \frac{\alpha^j (1+\rho)^j}{(1+\rho)^{N_S}} \quad \text{or} \quad P_j = \binom{N_S}{j} \alpha^j \frac{1}{(1+\rho)^{N_S - j}}$$

or $$P_j = \binom{N_s}{j} \alpha^j (1-\alpha)^{N_s - j} \qquad \text{......Eq.47}$$

This is the well known Binomial Formula. It implies that servers or links or trunks are independent of one another.

We have defined $\rho = R_s t_h$. The parameter $R_s t_h$ does not by itself specify the average activity of a source. Therefore, parameter $R_s t_h$ cannot be measured directly. So we can observe it as the average transmission rate per subscriber based on the total traffic.

It is defined as

$$\alpha' = \frac{A}{N_S} \qquad \text{Eq.48}$$

But A is defined as

$$A = R_S t_h (N_S - A_0) = \rho(N_S - A_0), \text{ since } \rho = R_S t_h \qquad \text{Eq.49}$$

Substituting Eqn. 49 in Eqn. 48 , we get

$$\alpha' = \frac{A}{N_S} = \frac{\rho(N_S - A_0)}{N_S} = \rho\left(1 - \frac{A_0}{N_S}\right) \qquad \text{Eq.50}$$

But A_0 is defined as

$$\text{GOS} = \frac{A - A_0}{A} \quad \text{or} \quad \text{GOS}\, A = A - A_0$$

or $A_0 = A(1 - \text{GOS})$ Eq.51

Substituting Eqn. 51 in Eqn. 50 , we get

$$\alpha' = \rho\left[1 - \frac{A}{N_S}(1 - \text{GOS})\right]$$

On simplification of above eqn, we get

$$\alpha' = \frac{\rho}{1 + \rho(1 - \text{GOS})} \qquad \text{...... Eq.52}$$

For Bernaulli traffic, grade of service (GOS) is zero. Therefore Eqn. 52 can be written as

$$\alpha' = \frac{\rho}{1+\rho} \qquad \text{...... Eq.53}$$

But we already studied that

$$\alpha = \frac{\rho}{1+\rho} \qquad \text{...... Eq.54}$$

Therefore, for this case both α and α' are same

Example 10.1: In a telephone system, there are 20,servers or links or trunks and 100 subscribers. On an average, 10 servers are busy at any time. The probability of all the servers being busy is 0.2.

Determine the GOS for two cases

(a) Erlang traffic

(b) Engest traffic.

Solution:

(a) For Erlang traffic,

GOS is the same as P_{NL}, Therefore, GOS = 0.2

(b) For Engest

$$GOS = \frac{N_S - N_L}{N_s - A_o} P_{NL}$$

Given N_s = 100 = No. of subscribers

N_L = 20 = No. of services

A_o = Carried traffic = No. of average busy servers = 10

P_{NL} = 0.2

$$GOS = \frac{100 - 20}{100-10} * 0.2$$

$$= 0.1875$$

GOS for Erlang traffic = 0.2

GOS for Engest traffic = 0.1875

As expected, the GOS is lower in the case of LCC model with finite sources.

Lost Calls Returned (LCR) System

In the lost calls cleared (LCC) model, we have assumed that unserviceable requests leave the system and never return. In other words, we can say that the call arrival rate into system is not affected by the rejected calls. This is usually not the case, particularly in subscriber concentrator system, calls to busy lines etc. Rejected calls in such cases do return to the system in the form of retries. Therefore, offered traffic now comprises of two components:

Offered traffic = New traffic + Retry traffic

Here, we derive the blocking probability P_B relationships taking into account the returning calls. The model which is used for analysis is called Lost Calls Returned (LCR) model.

In this analysis, we make the following assumptions regarding to the nature of the returning calls.

1. No new call is originated when a blocked call is being retried.
2. A number of retry attempts may be done before a call eventually gets serviced.
3. Retries are attempted after a random time and each retry time is statistically independent of the other.
4. Typical waiting time before a retry is longer than the average holding time.

Assumption (3) says that the retries are not correlated. Traffic peaks intervals would complicate the analysis.

Assumption (4) permits the system to maintain its statistical equilibrium even in the presence of retry.

We also assume that the waiting time is longer than the average service time, the retry arrivals are made to have the same arrival characteristic as the new traffic.

We can say that by using assumptions mentioned above the retry traffic is statistically indistinguishable from the new arrivals. Hence blocked cases merely add to the first attempt calls. We let R be the call arrival rate for new calls and we denote the GOS as P_C, call congestion. Then P_C calls are rejected. These rejected calls return to the system as retries. The retries will further experience blocking by factor of P_C, i.e. P_C x P_C calls will be rejected.

Effective call arrival rate is given by,

$$R_E = R + P_C R + P^2_C R + P^3_C R + \dots\dots\dots$$

$$= \frac{R}{1 - P_C} = \frac{R}{1 - GOS} \qquad \dots\dots\dots Eq.55$$

Eqn.55 gives the relationship between the effective call arrival rate R_E and call congestion P_C or GOS. In this case, we cannot directly determine either R_E or P_C as one is expressed in terms other. They are estimated iteratively using Erlang B formula. We state with R as the initial value for the call arrival rate. We have used same procedure for the LCC model with finite sources.

The effect of returning traffic is insignificant when operating at low GOS values. The effect of returning traffic is noticeable as high GOS values.

The estimated blocking probability has lower value in the case of LCC model than in the case LCR for a given traffic load and the system capacity. For a given value of GOS, LCC model permits a larger offered load to the system than the LCR model.

Lost Calls Held (LCH) System

Lost calls held (LCH) systems are distinctly different from delay systems. In delay systems, the messages are queued and taken up for service as and when resources are available. The total call time in a delay system is the sum of the waiting time and the service time. But, in the LCH model, the total time spent in the system is independent of the waiting time.

The total time spent in a system is determined by the average service time required. When a call arrives in a system it requires service continuously for a period of time and terminates after that time irrespective of whether it is being actually serviced or not. If a call is blocked, a portion of it is lost until server or link becomes free to service the call. It is illustrated in Fig.10.1.

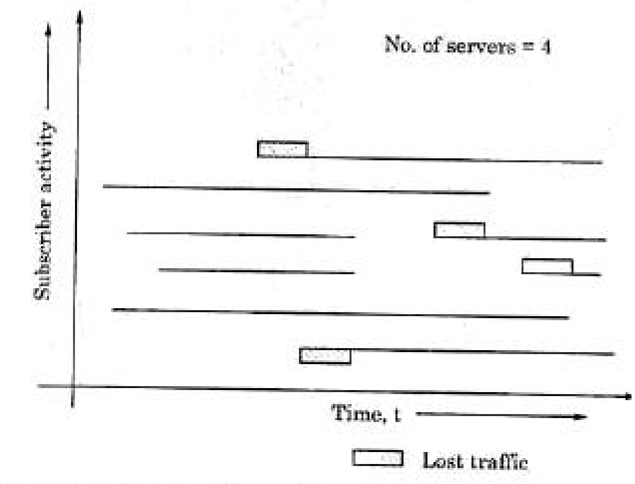

Fig.10.1 Illustration of lost traffic in LCH Model.

An example of LCH system is the time assigned speech interpolation (TASI) system. In a TASI system, the number of conversations supported is larger than the number of transmission channels or servers or links in the system.

This system exploits the fact that a speech conversation is interspersed with silence or inactivity period when there is no need for transmitting the signal. The transmission channel is deassigned during the silence period and the same channel is used for supporting another conversation which is in the activity phase. Therefore, more number of conversations is possible than the transmission channels can be supported on the system.

If a speech circuit becomes active when all the channels are busy, it is blocked and speech clipping occurs. A speech segment becomes active or inactive irrespective of whether a channel is available or not.

In other words, the duration for which a source is active is independent of whether it is being serviced or not.

As soon as a call arrives, it is accepted by the system whether a server or link is available or not. Hence, the number of active sources in the system is equal to the number of call arrivals in holding time t_h. For an infinite population the number of active sources in the system is characterized by Poisson arrival process.

Example10.2 A time assigned speech interpolation (TASI) system has 10 transmission channels and 20 sources connected to it. Find the probability of clipping if the activity factor for each source is 0.4.

Solution: Clipping occurs if 10 or more sources are in the activity phase simultaneously. Therefore, the probability that 10 or more sources are active gives the clipping probability.

This can be determined by using Poisson equation as:

Probability of clipping

$$= \sum_{j=10}^{\infty} P_j(t=8) = 1 - \sum_{j=0}^{9} P_j(t=8)$$

$$= 1 - e^{8}\left(1 + \frac{8}{\underline{\angle 1}} + \frac{8^2}{\underline{\angle 2}} + \frac{8^3}{\underline{\angle 3}} + \dots + \frac{8^9}{\underline{\angle 9}}\right) = 0.284$$

Probability of clipping = 0.284